

PROBLEM SET 5

[J]=Jacobson

- (1) [J] p. 256 #1
- (2) [J] p. 256 #2
- (3) [J] p. 260 #2
- (4) [J] p. 260 #7
- (5) [J] p. 266 #1(a)
- (6) [J] p. 266 #8
- (7) [J] p. 266 #9
- (8) Suppose that $f = x^n + px + q$. Show that $\lambda_1 = \lambda_2 = \cdots = \lambda_{n-2} = 0$, $\lambda_{n-1} = -(n-1)p$, $\lambda_n = -nq$, $\lambda_{n+1} = \cdots = \lambda_{2n-3} = 0$, and $\lambda_{2n-2} = (n-1)p^2$. Hence show the discriminant Δ of f is $\eta_{n+1}n^nq^{n-1} - \eta_n(n-1)^{n-1}p^n$, where $\eta_n = 1$ if $n \equiv 0, 1 \pmod{4}$ and $\eta_n = -1$ if $n \equiv 2, 3 \pmod{4}$. [Hint: use Newton's identities on p. 140 of Jacobson.]
- (9) Determine Galois groups (over \mathbb{Q}) of
 - (a) $x^4 + 8x - 12$,
 - (b) $x^4 + 1$,
 - (c) $x^4 + x^3 + x^2 + x + 1$,
 - (d) $x^4 - 2$.