Hand in all.

(1) [Jacobson, p. 287 #1] Show that \( \sin(u) \) is transcendental for all algebraic \( u \neq 0 \).

(2) Suppose \( L/K \) is an extension, and that \( L \) is finitely generated over \( K \). Show that the field \( K_a \) of elements of \( L \) which are algebraic over \( K \) is f.g. over \( K \).

(3) Suppose that \( p \) is prime and doesn’t divide \( m \), and let \( \epsilon \) be a primitive \( m^{th} \) root of 1 over \( \mathbb{Z}_p \). Show that \([\mathbb{Z}_p(\epsilon) : \mathbb{Z}_p] = k\), where \( k \) is the order of \( \bar{p} \) in \( \mathbb{Z}_m^* \). Show that the cyclotomic polynomial \( \Phi_m \) is irreducible over \( \mathbb{Z}_p \) if and only if \( \mathbb{Z}_m^* \) is a cyclic group generated by \( \bar{p} \). When is \( \Phi_4 \) irreducible over \( \mathbb{Z}_p \)? How about \( \Phi_8 \)?

(4) Determine whether \( \Phi_{18} \) is irreducible over \( \mathbb{Z}_{23}, \mathbb{Z}_{43}, \text{ and } \mathbb{Z}_{73} \). (You may want to look at I.L.22 and the comments after it.)

(5) Show that the primitive \( n^{th} \) roots of 1 over \( \mathbb{Q} \) form a normal basis for the splitting field of \( x^n - 1 \) over \( \mathbb{Q} \) if and only if \( n \) has no repeated prime factors.

(6) [Jacobson, p. 300 #1] Show that if \( E \) is a finite field and \( F \) is a subfield, so that \( E/F \) is a cyclic extension, then the norm homomorphism \( N_{E/F} \) of \( E^* \) is surjective on \( F^* \).

(7) [Jacobson, p. 305 #3] Show that if \( G \) is a transitive subgroup of \( S_n \) containing an \((n-1)\)-cycle and a transposition, then \( G = S_n \).

(8) [Jacobson, p. 305 #4] Find \( \text{Gal}_Q(f) \) for \( f(x) = x^6 - 12x^4 + 15x^3 - 6x^2 + 15x + 12 \).