

PUTNAM PRACTICE PROBLEMS
FOR SEPT. 23, 2016 - SUMS

EASY :

$$\sum_{n=1}^N n, \sum_{n=1}^N n^2, \sum_{n=1}^N n^3, \sum_{n=1}^N \frac{n-1}{n!}$$

$$\sum_{n=1}^N n \cdot n!, \sum_{n=1}^N \frac{1}{n(n+1)}, \sum_{n=1}^{\infty} \frac{n}{2^n}$$

$$\sum_{n=1}^{\infty} \frac{n(n+1)}{2^n}, \sum_{n=1}^N n 2^n$$

MEDIUM :

$$\sum_{n=1}^{\infty} \frac{2n+3}{n^2(n+1)^2(n+2)^2(n+3)^2}$$

$$\sum_{n=1}^{\infty} \frac{2n+3}{n(n+1)^2(n+2)^2(n+3)}$$

$$\sum_{n=1}^N \frac{n}{n^4 + n^2 + 1}$$

$$\prod_{n=2}^{\infty} \left(1 - \frac{1}{n^2}\right), \prod_{n=2}^{\infty} \frac{n^3-1}{n^3+1}$$

$$1 - \frac{1}{4} + \frac{1}{7} - \frac{1}{10} + \frac{1}{13} - \dots$$

$$\sum_{n=2}^N h(n-1) \binom{N}{n}$$

INTERESTING: (1) $\sum_{n=0}^{\infty} \frac{x^{(2^n)}}{1-x^{(2^{n+1})}}$ ($0 < x < 1$)

(2) $\prod_{n=1}^{\infty} \frac{1 + 2 \cos \frac{2x}{3^n}}{3}$

(3) $\lim_{n \rightarrow \infty} \sum_{k=n}^{2^n} \frac{1}{k}$

(4) $\lim_{n \rightarrow \infty} \frac{1}{n} \left[\frac{1}{\sqrt[n]{1}} + \frac{1}{\sqrt[n]{2}} + \frac{1}{\sqrt[n]{3}} + \dots + \frac{1}{\sqrt[n]{n}} \right]$

(5) $\sum_{n=0}^{\infty} \arccot(n^2 + n + 1)$

(6) $\lim_{n \rightarrow \infty} \frac{1}{n^4} \prod_{i=1}^{2n} (n^2 + i^2)^{\frac{1}{n}}$

(7) Let $u = 1 + \frac{x^3}{3!} + \frac{x^6}{6!} + \frac{x^9}{9!} + \dots$,

$v = \frac{x}{1!} + \frac{x^4}{4!} + \frac{x^7}{7!} + \dots$, $w = \frac{x^2}{2!} + \frac{x^5}{5!} + \frac{x^8}{8!} + \dots$

Show that $u^3 + v^3 + w^3 - 3uvw = 1$

(8) $\sum_{n=1}^{\infty} \frac{6^n}{(3^{n+1} - 2^{n+1})(3^n - 2^n)}$

(8) If $(m, n) = d$ means that the g.c.d. of m and n is d , and given that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$

show that $\sum_{m, n=1}^{\infty} \frac{1}{(mn)^2}$ is rational

(9) $\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{m^2 n + n^2 m + 2mn}$ ($(m, n) = 1$)