1. Prove the squeeze theorem: Let $I$ be an open interval in $\mathbb{R}$, and $a \in I$. Assume that
\[ f(x) \leq g(x) \leq h(x) \quad \forall x \in I \setminus \{a\}, \]
and
\[ \lim_{x \to a} f(x) = \lim_{x \to a} h(x). \quad (1.1) \]
Prove that $\lim_{x \to a} g(x)$ exists and equals the limit in (1.1).

2. Suppose $(a_n)$ and $(b_n)$ are convergent sequences. Prove that
\[ \lim_{n \to \infty} a_n b_n = \left( \lim_{n \to \infty} a_n \right) \left( \lim_{n \to \infty} b_n \right). \]

3. Prove or disprove: if the sequence $(a_n + b_n)$ converges, then both $(a_n)$ and $(b_n)$ converge.

4. Suppose $I$ is an interval in $\mathbb{R}$, $(x_n)$ is a sequence in $I$ that converges to $c \in I$. Suppose $f : I \to \mathbb{R}$. Prove that if $f$ is continuous at $c$, then
\[ \lim_{n \to \infty} f(x_n) = f(c). \]

5. Give an example to show that the converse to the assertion in (4.) is not true.

6. Give an example of a continuous bounded function on a closed interval that does not attain its maximum.