1. Suppose $I$ is an open interval in $\mathbb{R}$, that $f : I \rightarrow \mathbb{R}$, and $c \in I$. Suppose that for every sequence $(x_n)$ in $I$ that converges to $c$, $\lim_{n \to \infty} f(x_n)$ exists. Prove that all the limits are equal, and that $\lim_{x \to c} f(x)$ exists and equals the same number.

2. Prove that
$$\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}.$$

3. Prove that the equation
$$x^2 = x \sin x + \cos x$$
has exactly two real solutions.

4. Prove that $| \sin x - \sin y | \leq | x - y |$, for all $x, y \in \mathbb{R}$.

5. A number $c$ is called a zero of multiplicity $m$ for the polynomial $p$ if $p(x) = (x - c)^mq(x)$, where $q$ is a polynomial and $q(c) \neq 0$. Suppose $p$ has $r$ zeros in the interval $[a, b]$, counting each zero as often as its multiplicity. Prove that $p'$ has at least $r - 1$ zeros in $[a, b]$.

6. On HW 4, you did exercise 4.7 in Structure and Proof. Use this to prove the binomial theorem, exercise 4.8.