204: Homework 13 Due Tuesday April 26

1. Evaluate $\int\int_\Sigma xydS$, where Σ is the triangular region with vertices $(1,0,0),(0,2,0)$, and $(0,0,2)$.

2. Evaluate $\int\int_\Sigma ydS$, where Σ is the surface $z = \frac{2}{3}(x^{3/2} + y^{3/2})$, and $0 \leq x \leq 1, 0 \leq y \leq 1$.

3. Evaluate $\int\int_\Sigma x^2 + y^2 + z^2 dS$, where Σ is the part of the cylinder $x^2 + y^2 = 9$ between the plane $z = 0$ and $z = 2$, including the top and bottom faces.

4. Evaluate $\int\int_\Sigma \vec{F} \cdot d\vec{S}$, where $\vec{F}$ is the vector field $x\mathbf{i} + y\mathbf{j} + z^4\mathbf{k}$ and Σ is the part of the cone $z = \sqrt{x^2 + y^2}$, beneath the plane $z = 1$ with downward orientation.

5. Evaluate $\int\int_\Sigma \vec{F} \cdot d\vec{S}$, where $\vec{F}$ is the vector field $xze^y\mathbf{i} + -xze^y\mathbf{j} + z\mathbf{k}$ and Σ is the part of the plane $x+y+z = 1$ in the first octant with downward orientation.

6. A fluid has density $900kg/m^3$ and flows with velocity $\mathbf{v} = z\mathbf{i}+y^2\mathbf{j}+x^2\mathbf{k}$, where distance is measured in meters and time in seconds. Find the rate of flow outward through the cylinder $x^2 + y^2 = 4, 0 \leq z \leq 1$.

7. Use Gauss's law ($Q = \varepsilon_0 \int\int_\Sigma \vec{E} \cdot d\vec{S}$) to calculate the charge enclosed by the cube with vertices $(\pm1, \pm1, \pm1)$ if $\vec{E}$ is $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.