204: Homework 8 Due March 22

1. Consider the function $f(x, y) = x^3 + e^{3y} - 3xe^y$.
(a) Show that f has exactly one critical point $\mathbf{a}$.
(b) Show that a is a local minimum point.
(c) Is a a global minimum point?

2. The materials for the sides of a rectangular box cost twice as much as the material for the top and bottom. Find the relative dimensions of the box with greatest volume that can be constructed for a given cost.

3. (a) Find the minimum value of $f(x, y) = x^2+y^2$ on the curve $x+y = 2$. Why is there no maximum?
(b) Find the maximum value of $g(x, y) = x + y$ on the curve $x^2 + y^2 = 2$. Is there a minimum?
(c) How are parts (a) and (b) related?

4. A wire has the shape of the circle $x^2 + y^2 - 2y = 0$. Its temperature is given by $T(x, y) = 2x^2 + 3y$. Find the maximum and minimum temperature of the wire.

5. The temperature is given by $f(x, y, z) = 3xy + z^3 - 3z$. Prove that there are hottest and coldest points on the sphere $x^2 + y^2 + z^2 = 3$ and find them.

6. Suppose $x_1, x_2, \ldots, x_n$ are positive numbers. Prove the arithmetic-geometric inequality:

$$\sqrt[n]{x_1 x_2 \cdots x_n} \leq \frac{x_1 + x_2 + \cdots + x_n}{n}.$$