204: Homework 9 Due March 29

1. Calculate the line integral $\int F \cdot d\alpha$ along the given path:

(i) $F(x,y) = (x^2 - 2xy)\mathbf{i} + (y^2 - 2xy)\mathbf{j}$, along the parabola $y = x^2$ between $(-1,1)$ and $(1,1)$.

(ii) $F(x,y,z) = x\mathbf{i} - 2z\mathbf{j} + y\mathbf{k}$, and $\alpha(t) = 2t\mathbf{i} + 4t\mathbf{j} - t^2\mathbf{k}$, for $0 \le t \le 1$.

2. Calculate the work done by the force $F(x,y) = (x^2 - y^2)\mathbf{i} + 2xy\mathbf{j}$ in moving a particle in the counterclockwise direction around the square with corners $(0,0), (a,0), (a,a), (0,a)$. (The work is the line integral of the force).

3. Prove that the following two fields are not conservative by evaluating appropriate partial derivatives. Then for each one, find a closed path C such that the line integral around C is not 0.

(i) $F(x,y) = y\mathbf{i} - x\mathbf{j}$.

(ii) $F(x,y) = y\mathbf{i} + (xy - x)\mathbf{j}$.

4. For each of the following vector fields, determine whether they are conservative. If they are, find a potential function.

(i) $F(x,y) = x\mathbf{i} + y\mathbf{j}$.

(ii) $F(x,y) = 3x^2y\mathbf{i} + x^3\mathbf{j}$.

(iii) $F(x,y) = (2xe^y + y)\mathbf{i} + (x^2e^y + x - 2y)\mathbf{j}$.

(iv) $F(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

(v) $F(x,y,z) = (x+z)\mathbf{i} - (y+z)\mathbf{j} + (x-y)\mathbf{k}$.

5. A fluid flows in the xy-plane, each particle moving directly away from the origin. If a particle is a distance r from the origin, its speed is ar^b, where a and b are constants.

(i) Determine those values of a and b for which the velocity vector field is the gradient of a scalar field on $\mathbb{R}^2 \setminus \{0\}$.

(ii) When the velocity field is a gradient, find a potential function. (The case $b = -1$ should be treated separately).