1. (i) Consider the 1-dimensional system $\dot{x} = f(x)$ where $f$ looks like the picture. What are the fixed points, and which ones are attracting, and which ones are repelling?

(ii) If $f$ is changed to $f(x) + \varepsilon$, where $\varepsilon$ is a small real number (which may be positive or negative) how does the answer change?

2. (i) Sketch the trajectories of a 2-dimensional linear system whose fixed point is a center.
   (ii) What could happen qualitatively if there were higher order non-linear terms?
   (iii) If you knew that the system was conservative, how would that change the answer to (ii)?

3. (i) Consider the 2-dimensional system

\[
\begin{align*}
\dot{x} & = x + 2y \\
\dot{y} & = 3x + 4y
\end{align*}
\]

What is the nature of its fixed point?
(ii) What could happen qualitatively to the fixed point at 0 if one looked at the system

\begin{align*}
\dot{x} &= x + 2y + xy \\
\dot{y} &= 3x + 4y - y^2
\end{align*}

4. (i) What is the index of a closed curve with respect to a vector field \( \vec{F}(x, y) \)?
(ii) What is the index of the vector field

\[ \vec{F}(x, y) = \begin{pmatrix} x^2 \\ x - y \end{pmatrix} \]

at (0, 0)?

5. Consider the damped harmonic oscillator

\[ \ddot{x} + bx + x = 0 \]

where \( b \geq 0 \). Analyze the system and describe it qualitatively as \( b \) ranges from 0 to \( +\infty \).