1. Prove that \( \forall z_1, z_2 \in \mathbb{C} \), the following identities hold:
\[
|z_1 z_2| = |z_1| |z_2|, \quad \bar{z}_1 + \bar{z}_2 = \overline{z_1 + z_2}, \quad \bar{z}_1 \bar{z}_2 = \bar{z}_1 z_2.
\]

2. Prove that if \( z \) is a root of the polynomial \( p \in \mathbb{R}[x] \), then so is \( \bar{z} \).

3. Prove that if \( z_1, \ldots, z_n \in \mathbb{C} \), then \( |z_1 + \ldots + z_n| \leq |z_1| + \ldots + |z_n| \). When is equality obtained?

4. Prove that if \( z_1, z_2 \in \mathbb{C} \), then \( |z_1 - z_2| \geq |z_1| - |z_2| \). When is equality obtained?

5. Prove that the sum of the squares of the lengths of the diagonals of a parallelogram equals the sum of the squares of the lengths of the sides.

6. Prove that if \( z_1, z_2, z_3 \) are non-collinear points in the complex plane, the medians of the triangle with vertices \( z_1, z_2, z_3 \) intersect at the point \( \frac{1}{3}(z_1 + z_2 + z_3) \). What geometric fact does this prove?