416: Homework 1 Due: 5/September

1. Prove that $\forall\ z_1, z_2 \in \mathbb{C}$, the following identities hold:

$$|z_1 z_2| \ = \ |z_1||z_2|, \ \bar{z}_1 + \bar{z}_2 \ = \ \overline{z_1 + z_2}, \ \ \overline{z_1 z_2} \ = \ \bar{z}_1 \bar{z}_2.$$

2. Prove that if z is a root of the polynomial $p \in \mathbb{R}[x]$, then so is $\bar{z}$.

3. Prove that if $z_1, \ldots z_n \in \mathbb{C}$, then $|z_1 + \ldots + z_n| \leq |z_1| + \ldots + |z_n|$. When is equality obtained?

4. Prove that if $z_1, z_2 \in \mathbb{C}$, then $|z_1 - z_2| \geq |z_1| - |z_2|$. When is equality obtained?

5. Prove that the sum of the squares of the lengths of the diagonals of a parallelogram equals the sum of the squares of the lengths of the sides.

6. Prove that if z_1, z_2, z_3 are non-collinear points in the complex plane, the medians of the triangle with vertices z_1, z_2, z_3 intersect at the point $\frac{1}{3}(z_1 + z_2 + z_3)$. What geometric fact does this prove?