1. (i) Prove that the Laplacian can be written as
\[ \Delta = 4 \frac{\partial^2}{\partial z \partial \bar{z}}. \]
(ii) Suppose \( f \) is holomorphic. What is \( \Delta |f(z)|^2 \)?

2. Suppose \( u \) is a real-valued harmonic function on an open disk \( D \), and \( v \) is a harmonic conjugate of \( u \). Prove that \( uv \) and \( u^2 - v^2 \) are harmonic.

3. Prove that if \( u \) is a real-valued harmonic function, then \( \frac{\partial u}{\partial z} \) is holomorphic.

4. Suppose \( u \) is real-valued harmonic on an open disk \( D \), and so is \( u^2 \). Prove that \( u \) is constant.

5. Prove that if \( z_1, z_2, z_3 \) and \( w_1, w_2, w_3 \) are both triples of distinct points in \( \mathbb{C} \), then there exists a linear fractional transformation \( \phi \) such that \( \phi(z_j) = w_j \) for \( j = 1, 2, 3 \). Prove that \( \phi \) is unique.