1. Does \( \log |z(z - 1)| \) have a harmonic conjugate in \( \mathbb{C} \setminus [0, 1] \)? What about \( \log |(z - 1)/z| \)?

2. Prove that a positive harmonic function on \( \mathbb{C} \) is constant.

3. Locate and classify the isolated singularities of the following functions (do not ignore \( \infty \)).

\[
\frac{z^4}{z^2 - 4z + 3} \quad \frac{1}{\sin^3 z} \quad \sin \frac{1}{z}.
\]

4. Prove that a rational function can be decomposed into a sum of rational functions each of which has only one pole.

5. Prove that a function \( f \) is rational if and only if it is holomorphic on \( \mathbb{C} \setminus F \), where \( F \) is a finite set, and none of the singularities of \( f \) are essential (including the point at infinity).

6. Suppose that the holomorphic functions \( f \) and \( g \) both have poles of the same order at \( z_0 \). Prove

\[
\lim_{z \to z_0} \frac{f(z)}{g(z)} = \lim_{z \to z_0} \frac{f'(z)}{g'(z)}.
\]