416: Homework 9 Due: 14/November

1. Does $\log|z(z-1)|$ have a harmonic conjugate in $\mathbb{C}\setminus[0,1]$? What about $\log|(z-1)/z|$?

2. Prove that a positive harmonic function on $\mathbb{C}$ is constant.

3. Locate and classify the isolated singularities of the following functions (do not ignore ∞).

$$\frac{z^4}{z^2-4z+3} \qquad \frac{1}{\sin^3 z} \qquad \sin\frac{1}{z}.$$

4. Prove that a rational function can be decomposed into a sum of rational functions each of which has only one pole.

5. Prove that a function f is rational if and only if it is holomorphic on $\mathbb{C}\setminus F$, where F is a finite set, and none of the singularities of f are essential (including the point at infinity).

6. Suppose that the holomorphic functions f and g both have poles of the same order at z_0. Prove

$$\lim_{z\to z_0}\frac{f(z)}{g(z)} = \lim_{z\to z_0}\frac{f'(z)}{g'(z)}.$$