Math 131, Exam 3, November 13th

This exam should have 16 multiple choice questions, and two hand-written questions. Each multiple-choice question is worth 5 points; each hand-written problem is worth 10 points.

Mark your ID number on the six blank lines on the top of your answer card, using one line for each digit. Print your name on the top of the card.

Write your name and student ID number on each of the hand-graded sheets.

Mark your answer card with a **PENCIL** by shading in the correct box.

You may not use a calculator. There is a table of functions on page 2.

You may not have any written aids.

Table of functions

x	$\sin x$	$\cos x$	$\tan x$	$\ln x$	e^x
0	0	1.0	0		1.0
0.1	0.100	0.995	0.100	-2.303	1.105
0.2	0.199	0.980	0.203	-1.609	1.221
0.3	0.296	0.955	0.309	-1.204	1.350
0.4	0.389	0.921	0.423	-0.916	1.492
0.5	0.479	0.878	0.546	-0.693	1.649
0.6	0.565	0.825	0.684	-0.511	1.822
0.7	0.644	0.765	0.842	-0.357	2.014
0.8	0.717	0.697	1.030	-0.223	2.226
0.9	0.783	0.622	1.260	-0.105	2.460
1	0.841	0.540	1.557	0.000	2.718
1.1	0.891	0.454	1.965	0.095	3.004
1.2	0.932	0.362	2.572	0.182	3.320
1.3	0.964	0.267	3.602	0.262	3.669
1.4	0.985	0.170	5.798	0.336	4.055
1.5	0.997	0.071	14.101	0.405	4.482
1.6	1.000	-0.029	-34.233	0.470	4.953
1.7	0.992	-0.129	-7.697	0.531	5.474
1.8	0.974	-0.227	-4.286	0.588	6.050
1.9	0.946	-0.323	-2.927	0.642	6.686
2	0.909	-0.416	-2.185	0.693	7.389
2.1	0.863	-0.505	-1.710	0.742	8.166
2.2	0.808	-0.589	-1.374	0.788	9.025
2.3	0.746	-0.666	-1.119	0.833	9.974
2.4	0.675	-0.737	-0.916	0.875	11.023
2.5	0.598	-0.801	-0.747	0.916	12.182
2.6	0.516	-0.857	-0.602	0.956	13.464
2.7	0.427	-0.904	-0.473	0.993	14.880
2.8	0.335	-0.942	-0.356	1.030	16.445
2.9	0.239	-0.971	-0.246	1.065	18.174
3	0.141	-0.990	-0.143	1.099	20.086
3.1	0.042	-0.999	-0.042	1.131	22.198

1. Evaluate the derivative of $\sin(2x)$.

A. $\frac{1}{2}\sin(2x)$
B. $-\frac{1}{2}\sin(2x)$
C. $\frac{1}{2}\cos(2x)$
D. $-\frac{1}{2}\cos(2x)$
E. $2\sin(2x)$
F. $-2\sin(2x)$
G. $2\cos(2x)$
H. $-2\cos(2x)$
I. $\cos(2x)$
J. $-\cos(2x)$

$2\cos(2x)$

2. Let $p(x) = x^3 - 2x^2 + 5x - 4$. What is $p'(1)$?

A. -5
B. -4
C. -3
D. -2
E. -1
F. 1
G. 2
H. 3
I. 4
J. 5

$p'(x) = 3x^2 - 4x + 5$

$p'(1) = 4$

3. What is $\frac{d}{dx}\cot x$?

A. $\sec^2 x$
B. $-\sec^2 x$
C. $\csc^2 x$
D. $-\csc^2 x$
E. $\csc x \cot x$
F. $-\csc x \cot x$
G. $\sec x \tan x$
H. $-\sec x \tan x$
I. $\sec x \csc x$
J. $-\sec x \csc x$

$$\frac{d}{dx}\frac{\cos x}{\sin x} = \frac{-\sin x(\sin x) - \cos x(+\cos x)}{\sin^2 x}$$

$$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x}$$

$$= -\frac{1}{\sin^2 x} = -\csc^2 x$$

4. Let $f(x) = x^2 \ln x$. Which of the following is closest to $f'(2)$?

A. 0
B. 1
C. 2
D. 3
E. 4
F. 5
G. 6
H. 7
I. 8
J. 9

$$f'(x) = 2x \ln x + x^2 \cdot \frac{1}{x}$$

$$= 2x \ln x + x$$

$$f'(2) = 4 \ln 2 + 2$$

$$= 4(.693) + 2$$

$$\approx 4.8$$

$$\approx 5$$

5. What is the slope of the tangent line to the curve

$$x^2 + y^4 = 20$$

at the point $(2, 2)$?

A. -1/20
B. - 1/16
C. -1/8
D. - 1/2
E . -1
F. 1/20
G. 1/16
H. 1/8
I. 1/2
J. 1

$2x + 4y^3 y' = 0$

$y' = -\frac{x}{2y^3}$

$y'|_{(2,2)} = \frac{-2}{2 \cdot 2^3} = -1/8$

6. What is the slope of the tangent line to the curve

$$\sin y + x^2 = 1$$

at the point $(1, 0)$?

A. -2
B. -1
C. $-\sqrt{3}/2$
D. -1/2
E. 0
F. 1/2
G. $1/\sqrt{2}$
H. $\sqrt{3}/2$
I. 1
J. 2

$\cos y \, y' + 2x = 0$

$y' = -\frac{2x}{\cos y}$

$y'|_{(1,0)} = -2$

7. Let f, g and h be differentiable functions from $\mathbb{R}$ to $\mathbb{R}$. Here is a table of values for the functions and their derivatives:

x	$f(x)$	$g(x)$	$h(x)$	$f'(x)$	$g'(x)$	$h'(x)$
1.0	1.1	1.2	1.3	1.4	1.3	2
1.1	1.2	1.3	1.4	1.5	1.2	3
1.2	1.3	1.4	1.5	1.6	1.1	1
1.3	1.4	1.5	1.6	1.0	1.0	1
1.4	1.5	1.6	1.0	1.1	1.6	2
1.5	1.6	1.0	1.1	1.2	1.5	3
1.6	1.0	1.1	1.2	1.3	1.4	2

What is the derivative of $f \circ g \circ h$ at the point 1?

A. 1.6
B. 1.8
C. 2.0
D. 2.2
E. 2.4
F. 2.6
G. 2.8
H. 3.0
I. 3.2
J. 3.4

$h(1) = 1.3$

$g(1.3) = 1.5$

$f'(g \circ h(1))\, g'(h(1))\, h'(1) = f'(1.5)\, g'(1.3)\, h'(1)$

$= (1.2)(1.0)(2)$

$= 2.4$

8. With the same data as in Q. 7, what is the derivative of $\frac{f(x)}{g(x)}$ at $x = 1.5$?

A. - 2.0
B. - 1.8
C. -1.6
D. -1.4
E. -1.2
F. -1.0
G. -0.8
H. - 0.6
I. -0.4
J. -0.2

$$\frac{gf' - fg'}{g^2} = \frac{1.2 - (1.6)(1.5)}{1}$$

$= 1.2 - 2.4$

$= -1.2$

$1.6 \times 1.5 = 2.4$

9. A bacterial population P satisfies the differential equation

$$\frac{dP}{dt} = 1.5P, \qquad P(0) = 1000.$$

What is $P(2)$, to the nearest 1000?

A. 13,000
B. 14,000
C. 15,000
D. 16,000
E. 17,000
F. 18,000
G. 19,000
H. 20,000
I. 21,000
J. 22,000

$P(t) = P(0)e^{1.5t}$

$\therefore P(2) = 1000\, e^{3} = 20{,}086$

10. Which of these numbers is closest to $\sqrt{99}$?

A. 9.55
B. 9.60
C. 9.65
D. 9.70
E. 9.75
F. 9.80
G. 9.85
H. 9.90
I. 9.95
J. 10.0

$\sqrt{a+h} = (a+h)^{1/2}$

$\approx a^{\frac{1}{2}} + h\,\frac{1}{2}a^{-1/2}$

$\sqrt{100-1} \approx 10 + (-1)\left(\frac{1}{2}\right)\left(\frac{1}{10}\right)$

$10 - \frac{1}{20}$

$= 9.95$

11. Using the table of functions at the beginning of the exam, which of these numbers is closest to $\cos(1.61)$?

A. -.01
B. -.02
C. -.03
D. -.04
E. -.05
F. -.06
G. -.07
H. -.08
I. -.09
J. -.10

By tangent line approximation

$\cos(1.61) \approx \cos(1.6) - (.01)\sin(1.6)$

$= -.029 - (.01)(1)$

$= -.039$

12. Which of the following is closest to

$$\frac{d}{dx} e^{2x}\Big|_{x=0}?$$

A. 0
B. 1
C. 2
D. 3
E. 5
F. 7
G. 10
H. 14
I. 17
J. 20

$\frac{d}{dx} e^{2x} = 2e^{2x}$

$\frac{d}{dx} e^{2x}\Big|_{x=0} = 2e^{0} = 2$

13. To the nearest integer, what is

$$\lim_{n\to\infty}(1+\frac{2}{n})^n?$$

A. 0
B. 1
C. 2
D. 3
E. 4
F. 5
G. 6
H. 7
I. 8
J. 9

$e^r = \lim_{n\to\infty}\left(1+\frac{r}{n}\right)^n$

$\therefore$ Answer is $e^2 = 7.389$

14. Let

$$g(x) = \frac{x^2+1}{x-2}.$$

What is $g'(1)$?

A. -4
B. -3
C. -2
D. -1
E. 0
F. 1
G. 2
H. 3
I. 4
J. 5

$$g'(x) = \frac{(x-2)(2x)-(x^2+1)}{(x-2)^2}$$

$$= \frac{x^2-4x-1}{(x-2)^2}$$

$g'(1) = -4$

15. A spherical balloon is losing air at a rate of 10π cubic centimeters per minute. At what rate is its radius r decreasing when r is 2 centimeters?

A. .125 cm/min
B. .25 cm/min
C. .375 cm/min
D. .5 cm/min
E. .625 cm/min
F. .75 cm/min
G. .875 cm/min
H. 1.0 cm/min
I. 1.125 cm/min
J. .125 cm/min

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 \frac{dr}{dt}$

$\therefore r' = \frac{1}{4\pi r^2} V' = \frac{10\pi}{4\pi(4)}$

$= \frac{10}{16} = \frac{5}{8}$

16. Calculate

$$\frac{d}{dx}\arctan(x^2)|_{x=2}.$$

A. 0
B. 1/17
C. 1/5
D. 1/2
E. 2/17
F. 2/5
G. 1
H. 4/17
I. 4/5
J. 2

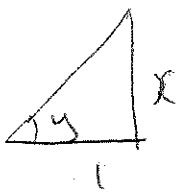
$\frac{d}{dx}\text{Arctan}(x) = ?$

$y = \text{Arctan}\, x$

$\text{Tan}\, y = x$

$\text{Sec}^2 y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\text{Sec}^2 y} = \text{Cos}^2 y = \frac{1}{1+x^2}$



$\therefore \frac{d}{dx}\text{Arctan}(x^2) = \frac{1}{1+x^4} \cdot 2x$

At $x = 2$, get $\frac{4}{17}$.

Math 131 Hand-graded Problems

Student Name:

Student ID:

Problem A: A cup of hot chocolate has temperature $80°C$ in a room kept at $20°C$. After 15 minutes the hot chocolate cools to $60°C$. What will its temperature be after another 15 minutes?

Let $u(t) = T(t) - 20$, where T is temperature of the cup.

$\frac{du}{dt} = ku$ and $u(0) = 60$.

$u(t) = 60e^{+kt}$

$u(15) = 40 = 60e^{15k}$

$\therefore e^{15k} = \frac{40}{60} = \frac{2}{3}$.

$u(30) = 60e^{30k} = 60(e^{15k})^2 = 60\left(\frac{2}{3}\right)^2$

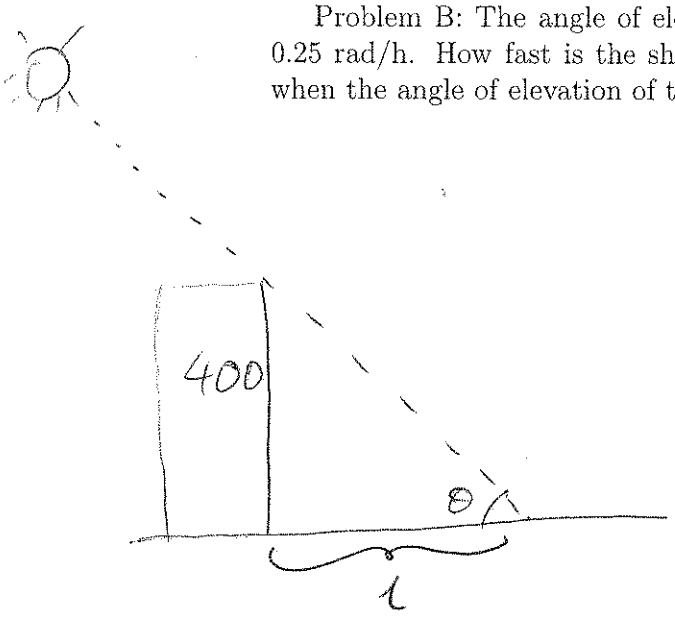
$= \frac{80}{3} = 26.\overline{6}$

$\therefore T(30) = 20 + u(30) = 46.\overline{6}\,°C$

Student Name:

Student ID:

Problem B: The angle of elevation of the sun is decreasing at a rate of 0.25 rad/h. How fast is the shadow cast by a 400 foot building increasing when the angle of elevation of the sun is $\pi/6$?



$$\cot\theta = \frac{l}{400}$$

where l = length of shadow

θ = angle of elevation of sun

$$\therefore l = 400\cot\theta$$

$$\frac{dl}{dt} = 400\frac{d}{d\theta}(\cot\theta)\frac{d\theta}{dt}$$

By Q.3, $\frac{d}{d\theta}\cot\theta = -\operatorname{cosec}^2\theta$. $\therefore \frac{dl}{dt} = -400\operatorname{cosec}^2\theta\frac{d\theta}{dt}$

When $\theta = \pi/6$, $\operatorname{cosec}^2\theta = \frac{1}{\sin^2\pi/6} = \frac{1}{(\frac{1}{2})^2} = 4$

$$\therefore \frac{dl}{dt} = -400(4)(-0.25) = 400 \text{ ft/h}.$$