Math 415 Assignment 11
Due Thursday, December 8, at the beginning of class

Question 1. Find the harmonic function in the square

$$D = \{0 < x < \pi, 0 < y < \pi\}$$

with boundary values

$$\begin{aligned} u_y(x,0) &= 0 \\ u_y(x,\pi) &= 0 \\ u(0,y) &= 0 \\ u(\pi,y) &= \frac{1}{2}(1+\cos(2y)). \end{aligned}$$

Question 2. Solve

$$\begin{aligned} u_{xx} + u_{yy} &= 1 \quad \text{for } x^2+y^2 < 1 \\ u &= 1 \quad \text{for } x^2+y^2 = 1. \end{aligned}$$

In the last two questions, we use polar coordinates (r,θ).

Question 3. Let u be a harmonic function in the disc $D = \{r < 2\}$ which satisfies

$$u = 3\sin(2\theta) + 1 \quad \text{for } r = 2.$$

Without finding an explicit formula for u, answer the following questions:

(a) Find the maximum value of u in $\overline{D}$.
(b) Find $u(0)$.

Question 4. Find the harmonic function in the disc $\{r < 1\}$ with boundary conditions

$$u = 1 + 3\sin(\theta) \quad \text{for } r = 1.$$