Math 415 Assignment 2
Due Thursday, September 15, at the beginning of class

Question 1. For each of the following PDEs, determine if it is elliptic, hyperbolic or parabolic.

(a) $u_{xx} - u_{xy} + 2u_{yy} = 0$.
(b) $u_{xx} + 4u_{xy} + 4u_{yy} + u_x - u = 0$.
(c) $u_{xx} + u_{xy} - u_x + 3u_y + 3u = 0$.
(d) $2u_{xx} - 4u_{xy} + u_{yy} = 0$.

Question 2. Find the regions in the xy plane where the equation

$$(1+y)^2 u_{xx} + 2xyu_{xy} + x^2 u_{yy} = 0$$

is elliptic, hyperbolic or parabolic.

Question 3. Consider the equation

$$u_x + 2tu_t = 0.$$

(a) Show that there does not exist a solution which satisfies the initial condition $u(x,0) = x$.
(b) Show that there exist infinitely many solutions which satisfy the initial condition $u(x,0) = 5$.

Question 4. By trial and error, find a solution of the diffusion equation $u_t = u_{xx}$ with the following initial conditions:

(a) $u(x,0) = x^3$.
(b) $u(x,0) = x^4$.

Question 5. Let D be a bounded region in the plane $\mathbb{R}^2$ with smooth boundary ∂D and let f be a continuous function on $\mathbb{R}^2$. Consider the Neumann problem

$$\Delta u = f \quad \text{in } D$$
$$\frac{\partial u}{\partial n} = 0 \quad \text{on } \partial D.$$

(a) Show that if the problem has a solution, then it has infinitely many solutions.
(b) Use Green's theorem to show that if the problem has a solution, then

$$\iint_D f(x,y)\,dxdy = 0.$$