Math 415 Assignment 4
Due Thursday, September 29, at the beginning of class

Let

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-p^2}\, dp.$$

You may use without proof that

$$\lim_{x\to\infty} \operatorname{erf}(x) = 1.$$

Question 1. Solve the diffusion equation $u_t = u_{xx}$ on the whole line with initial condition $u(x,0) = e^{2x}$.

Question 2. Solve the diffusion equation $u_t = u_{xx}$ on the half line $x > 0$ with boundary condition $u(0,t) = 0$ and initial condition $u(x,0) = e^{-x}$. Express your solution in terms of erf.

Question 3. A wave $f(x+ct)$ travels along a semi-infinite string $(0 < x < \infty)$ for $t < 0$. Find the vibrations $u(x,t)$ of the string for $t > 0$ if the end $x = 0$ is fixed.

Question 4. Let u be a solution of the diffusion equation $u_t = ku_{xx}$ for $0 < x < 1$ and $0 < t < 1$ with $u(0,t) = u(1,t) = 0$ and $u(x,0) = x(1-x)$. Show that $u(x,t) \le \frac{1}{2}$ for all $0 < x < 1$ and $0 < t < 1$.

Question 5. Let u be a solution of the diffusion equation $u_t = ku_{xx}$ for $-1 < x < 1$ and $0 < t < 1$ with boundary condition $u(-1,t) = u(1,t) = g(t)$ and initial condition $u(x,0) = \varphi(x)$. Show that if φ is an even function, then $u(x,t)$ is an even function in x for every t.

Hint: Consider $v(x,t) = u(x,t) - u(-x,t)$ and use uniqueness of solutions