Math 415 Assignment 5
Due Thursday, October 6,
at the beginning of class

Question 1. Solve

$$u_{tt} - c^2 u_{xx} = xt^2$$
$$u(x,0) = 0$$
$$u_t(x,0) = 0$$

on the whole line.

Question 2. Solve

$$u_{tt} - c^2 u_{xx} = c^2 \sin(x)$$
$$u(x,0) = \sin(x)$$
$$u_t(x,0) = c\cos(x)$$

on the whole line.

Question 3. A wave $f(x+ct)$ travels along a semi-infinite string ($0 < x < \infty$) for $t < 0$. Assume that $f(0) = 0$. Find the vibrations $u(x,t)$ of the string for $t > 0$ if the end $x = 0$ is free, that is, assume a Neumann condition $u_x(0,t) = 0$.

Question 4. Solve the diffusion equation with a source on the half line with homogeneous Dirichlet boundary condition

$$u_t - ku_{xx} = f(x,t)$$
$$u(0,t) = 0$$
$$u(x,0) = \phi(x)$$

for $0 < x < \infty$ and $0 < t < \infty$. To this end, use the method of reflection as well as the solution for the diffusion equation with a source on the whole line from class.

Question 5. Solve the diffusion equation with a source on the half line with inhomogeneous Dirichlet boundary condition

$$v_t - kv_{xx} = f(x,t)$$
$$v(0,t) = h(t)$$
$$v(x,0) = \phi(x).$$

Note that we already started this problem in class: Consider $u(x,t) = v(x,t) - h(t)$, and use the solution of the preceding question.