Math 415 Assignment 6
Due Tuesday, October 25, at the beginning of class

Question 1. Using separation of variables, solve the diffusion problem with mixed boundary conditions:

$$\begin{aligned} u_t &= k u_{xx} \quad (0 < x < l), \\ u(0,t) &= u_x(l,t) = 0, \\ u(x,0) &= \phi(x). \end{aligned}$$

Express your answer as a (finite) Fourier series, assuming that ϕ has a suitable (finite) Fourier expansion.

Question 2. Using separation of variables, show that the problem

$$\begin{aligned} t u_t &= u_{xx} + 2u \quad (0 < x < \pi) \\ u(0,t) &= u(\pi,t) = 0 \\ u(x,0) &= 0 \end{aligned}$$

has infinitely many solutions.

Question 3. Solve the damped wave problem

$$\begin{aligned} u_{tt} &= c^2 u_{xx} - r u_t \quad (0 < x < l), \\ u(0,t) &= u(l,t) = 0, \\ u(x,0) &= \phi(x), \\ u_t(x,0) &= \psi(x), \end{aligned}$$

where $0 < r < \frac{2\pi c}{l}$, assuming that ϕ and ψ are of the form

$$\begin{aligned} \phi(x) &= \sum_{n=1}^{N} a_n \sin\left(\frac{n\pi x}{l}\right) \\ \psi(x) &= \sum_{n=1}^{N} b_n \sin\left(\frac{n\pi x}{l}\right) \end{aligned}$$

(It may be helpful to look up the general solution of a linear second order ODE with constant coefficients.)