Math 415 Assignment 8
Due Thursday, November 10, at the beginning of class

Question 1. Consider a metal rod ($0 < x < l$), insulated along its sides but not at its ends, which is initially at temperature 1 throughout the rod. At time $t = 0$, both ends of the rod are plunged into a bath of temperature 0. Let $u(x,t)$ denote the temperature of the rod at position x and time t.

(a) Write the PDE for $u(x,t)$ with boundary and initial conditions.
(b) Find a formula for $u(x,t)$.

Question 2. Solve $u_{tt} = c^2 u_{xx}$ for $0 < x < \pi$ with boundary conditions $u_x(0,t) = u_x(\pi,t) = 0$ and initial conditions $u(x,0) = 0, u_t(x,0) = \cos^2(x)$ (*Hint: Look back to Section 4.2*).

Question 3. (a) Compute the full Fourier series of x^2 on $[-1,1]$.
(b) Assuming that series in (a) converges to x^2 pointwise on $(-1,1)$, use the result of (a) to compute

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}.$$

Question 4. Find the full Fourier series of e^x in $(-1,1)$ in complex and real form.

Question 5. Consider the differential operator $A = \frac{d^4}{dx^4}$ on $(0,l)$ with boundary conditions

$$X(0) = X'(0) = X''(l) = X'''(l) = 0.$$

(a) Show that A is hermitian (i.e. show that if f, g satisfy the boundary conditions, then $\langle Af, g\rangle = \langle f, Ag\rangle$).
(b) Show that A is positive semi-definite (i.e. show that $\langle Af, f\rangle \geq 0$ if f satisfies the boundary conditions).