Math 415  Assignment 8  
Due Thursday, November 10,  
at the beginning of class

**Question 1.** Consider a metal rod ($0 < x < l$), insulated along its sides but not at its ends, which is initially at temperature 1 throughout the rod. At time $t = 0$, both ends of the rod are plunged into a bath of temperature 0. Let $u(x, t)$ denote the temperature of the rod at position $x$ and time $t$.

(a) Write the PDE for $u(x, t)$ with boundary and initial conditions.
(b) Find a formula for $u(x, t)$.

**Question 2.** Solve $u_{tt} = c^2 u_{xx}$ for $0 < x < \pi$ with boundary conditions $u_x(0, t) = u_x(\pi, t) = 0$ and initial conditions $u(x, 0) = 0, u_t(x, 0) = \cos^2(x)$ *(Hint: Look back to Section 4.2).*

**Question 3.**
(a) Compute the full Fourier series of $x^2$ on $[-1, 1]$.
(b) Assuming that series in (a) converges to $x^2$ pointwise on $(-1, 1)$, use the result of (a) to compute
\[
\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}.
\]

**Question 4.** Find the full Fourier series of $e^x$ in $(-1, 1)$ in complex and real form.

**Question 5.** Consider the differential operator $A = \frac{d^4}{dx^4}$ on $(0, l)$ with boundary conditions
\[X(0) = X'(0) = X''(l) = X'''(l) = 0 .\]

(a) Show that $A$ is hermitian (i.e. show that if $f, g$ satisfy the boundary conditions, then $(Af, g) = (f, Ag)$).
(b) Show that $A$ is positive semi-definite (i.e. show that $(Af, f) \geq 0$ if $f$ satisfies the boundary conditions).