HW4Solutions

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4.5.8

The first thing to recognize is the the power function evaluated at $\theta = 30000$ equals the type 1 error probability, so $\alpha = 0.01$. Since this test is based on a standard normal distribution, c will involve the quantile $z_{0.99} = 2.326$. Next, evaluate power at $\theta = 35000$ to solve for n :

$$0.98 = P\left(\frac{\bar{X} - 30000}{5000/\sqrt{n}} > 2.326|\theta = 35000\right)$$

$$0.98 = P\left(\frac{\bar{X} - 35000}{5000/\sqrt{n}} > 2.326 + \frac{30000 - 35000}{5000/\sqrt{n}}|\theta = 35000\right)$$

$$0.98 = P\left(Z > 2.326 + \frac{30000 - 35000}{5000/\sqrt{n}}\right)$$

Then, since $z_{0.02} = -2.05375$ we have

$$-2.05375 = 2.326 + \frac{30000 - 35000}{5000/\sqrt{n}}$$

so that (rounding up to nearest integer)

$$n = \lceil 4.37975^2 \rceil = 20.$$

Note the c value is $2.326\frac{5000}{\sqrt{n}} + 30000$.

4.5.10

Just like 4.5.8, we have that $\alpha = 0.10$ based on the power function evaluated at $p = 1/2$. Then,

$$0.95 = P\left(\frac{\hat{p} - 1/2}{\sqrt{(1/2 \times 1/2)/n}} > 1.282 \mid p = 2/3\right)$$

$$0.95 = P\left(Z > \frac{1.282\sqrt{1/(4n)} + 1/2 - 2/3}{\sqrt{2/(9n)}}\right)$$

$$-1.645 = \frac{1.282\sqrt{1/(4n)} + 1/2 - 2/3}{\sqrt{2/(9n)}}$$

You can solve this quadratic equation to find $n = \lceil 8.498763^2 \rceil = 73$. Compare this to the pwr.p.test function in R which says $n = 75$:

```
library(pwr)
```

```
## Warning: package 'pwr' was built under R version 3.4.4
```

```
h<-ES.h(.66667,0.5)
 pwr.p.test(h=h,power = 0.95,n=NULL,sig.level=0.10,alternative="greater")
```

```
##
##      proportion power calculation for binomial distribution (arcsine transformatio
n)
##
##               h = 0.339844
##               n = 74.14976
##       sig.level = 0.1
##           power = 0.95
##     alternative = greater
```

4.6.6

```
d.300 <- c(284,279,289,292,287,295,285,279,306,298)
d.600 <- c(298,307,297,279,291,335,299,300,306,291)
t.test.stat<-(mean(d.300)-mean(d.600) - 0)/(sqrt(var(d.300)/10 + var(d.600)/10))
v.3 <- var(d.300)
v.6 <- var(d.600)
df <- ((v.3 / 10 + v.6 / 10)^2)/((1/9)*(v.3/10)^2 + (1/9)*(v.6/10)^2)
t.test.stat
```

```
## [1] -2.03404
```

```
qt(0.975, df)
```

```
## [1] 2.137746
```

```
qt(0.025, df)
```

```
## [1] -2.137746
```

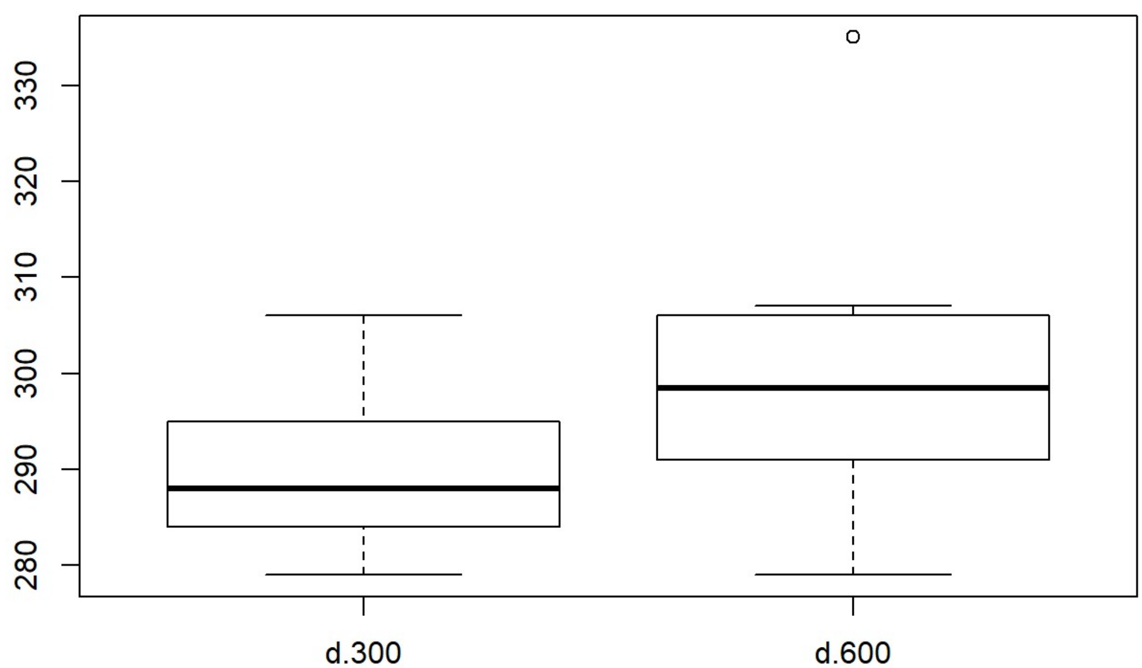
```
mean(d.300)-mean(d.600)
```

```
## [1] -10.9
```

```
ci<- c(mean(d.300)-mean(d.600)+qt(0.025, df)*(sqrt(var(d.300)/10 + var(d.600)/10)), me
an(d.300)-mean(d.600)+qt(0.975, df)*(sqrt(var(d.300)/10 + var(d.600)/10)))
ci
```

```
## [1] -22.3557409   0.5557409
```

```
boxplot(data.frame(cbind(d.300,d.600)))
```



The point estimate is -10.9. The CI is (-22.36, 0.56). The test of no difference in means is not rejected at $\alpha = 0.05$. We conclude there is not sufficiently strong evidence to reject the hypothesis that the two different doses of AZT result in equal average response among patients.

4.6.8

The hypotheses are $H_0 : p \leq 0.14$ and $H_1 : p > 0.14$. The critical value is $z_{0.99} = 2.326$ and the test statistic is $z = \frac{104/590 - 0.14}{\sqrt{.14 \times .86 \times 1/590}} = 2.54$. The p-value is $P(Z > 2.54) = 0.0055$. Reject H_0 .

4.6.10

Under H_0 we have

$$P(S_1^2/S_2^2 > F_{0.95}(12, 10)) = 0.05$$

so the cutoff value is $c = F_{0.95}(12, 10) = 2.913$.