Name: 

________________________________________________________

Please print above

Course: Math 127

Part of your name should be printed in large letters at the top of this page of your examination booklet. Your proctor can help you find your booklet if necessary. Make sure an answer card is on top of the booklet.

- make sure you have an adequate supply of PENCILS and ERASERS and your WASHINGTON UNIVERSITY photo ID card. Your proctors will check your ID card during the exam. Students without their ID will have their score withheld until they present their ID at the math office.

- PRINT your name and the course and exam number at the top of your card. Fill in your ID number in the appropriate boxes.

- Do not use any extra NOTES, BOOKS or SCRATCH PAPER. You should have ample space in your booklet for calculations. If you run out of space use the sides of the booklet pages for your work.

- CALCULATORS are only allowed if your instructor permits them.

- MARK your answer card neatly and make clean erasures. Sloppy cards will delay grading and result in your scores being withheld until you visit the math office to see your mismarkings.

- To see your exam score, go to the math department homepage at www.math.wustl.edu and use the link to 'Exam Scores' under 'Resources'.

- Scores on multiple choice questions will usually appear on the website within two days.

- Scores will also be posted by your course bulletin board in Cupples I.

For more information about your exam contact your instructor or the math department office in Cupples I, room 100.
PART I consists of 14 multiple choice questions (worth 5 points each), for a total of 70 points, and PART II consists of 6 true/false questions (worth 1 point each), for a total of 6 points. Mark the correct answer on the answer card. On PARTs I and II, only the answer on the card will be graded.

PART III consists of 3 questions (worth 8 points each), for a total of 24 points.

No calculators with a CAS are allowed.
PART I - Blacken your answers on the answer card.

1. Find $y'$ when $y = 2x^{-5}$.

A) $x' = \frac{-10}{x^6}$
B) $x' = \frac{2}{x^4}$
C) $x' = \frac{-2}{5x^4}$
D) $x' = \frac{-2}{5x^2}$
E) $x' = \frac{10}{x^5}$
F) $x' = \frac{10}{x^7}$
G) $x' = \frac{2}{x^{10}}$
H) $x' = \frac{2}{x^7}$
I) DNE
2. Solve for $x$: $\log_{b}x - \log_{b}25 = \log_{b}10^{18} - \log_{b}(10^{10} - x)$.

A) $b$
B) $b^{x}$
C) $25$
D) $10^{9}$
E) $10^{18}$
F) $10^{19}/25$
G) $5$
H) $5 \times 10^{9}$
I) No solutions exist.
3. A pen manufacturer determined that the cost in dollars of producing \( x \) dozen pens in one day is given by

\[ C(x) = 350 + 2x - 0.01x^2. \]

a.) Find the exact cost of producing 12 more pens at a production level of 70 dozen pens.

b.) Find the marginal cost at a production level of 70 dozen pens.

A) -14.81; -14.80

B) 0.59; 0.58

C) 0.59; 0.60

D) -179.04; -14.80

E) 5.76; 0.60

F) 0.60; 0.60

G) 0.58; 0.58

H) -14.80; -14.80

I) -14.82; -14.80
4. Find \( \lim_{x \to 5} \frac{x-5}{|x-5|} \).

A) 5
B) -5
C) 25
D) 0
E) 1
F) -1
G) DNE
H) \infty
I) -\infty
5. Find

\[ \lim_{x \to 4} \left( \frac{x^2 - 16}{x - 4} + \sqrt{x^2 - 16} \right) \]

A) \( \infty \)
B) \(-\infty\)
C) \( \pm \infty \)
D) 10
E) -10
F) 8
G) -8
H) 11
I) DNE
6. Refer to the graph of $f(x)$ below.

a.) Find $f(-3)$

b.) Find $\lim_{x \to -3} f(x)$.

c.) Is $f$ continuous at $x = -3$?

A) 3: 6; No

B) 6: 3; No

C) 6: 6; No

D) 3: 3; No

E) 6: -1; No

F) 3: 6; Yes

G) 6: 6; Yes

H) 3: 3; Yes

I) 6: 3; Yes
Let

\[ f(x) = \begin{cases} 
  x^2 - m & \text{if } x \leq 2, \\
  -x^2 + m & \text{if } x > 2
\end{cases} \]

For what values of \( m \), if any, is \( f \) continuous?

A) 2
B) -2
C) 1.99
D) 2.00001
E) 1
F) 4
G) -4
H) 0
I) \( f \) is discontinuous at \( x = 2 \)
8. Given the function

\[ f(x) = \begin{cases} 
-5 & \text{if } x \text{ is an odd integer,} \\
10 & \text{if } x \text{ is not an odd integer}
\end{cases} \]

a.) Evaluate \( \lim_{x \to 2} f(x) \),
b.) Evaluate \( f(2) \),
c.) Where is \( f \) discontinuous?

A) -5; 10; at odd integers
B) -5; -5; at odd integers
C) 10; 10; at all \( x \)
D) 10; 10; at odd integers
E) 10; 10; at even integers
F) 10; -5; at all \( x \)
G) -5; 10; at even integers
H) 10; -5; at even integers
I) DNE; 10; at odd integers
9. Solve for $x$:

$$\frac{x^2 + 40,000x}{x - 1.00001} > 0$$

A) $(0, 1.00001) \cup (40000, \infty)$

B) $(-\infty, 1.00001) \cup (0, \infty)$

C) $(-40000, 0) \cup (1.00001, \infty)$

D) $(-1.00001, 0) \cup (0, \infty)$

E) $(0, \infty)$

F) $(-\infty, 0) \cup (1.00001, \infty)$

G) $(-\infty, -40000)$

H) $(-\infty, -40000) \cup (0, \infty)$

I) $(-40000, \infty)$
10. Given \( f(x + h) - f(x) = 4xh + 4h + 2h^2 \), find the slope of the tangent line at \( x = 2 \).

A) 0
B) 2
C) 4
D) 6
E) 8
F) 10
G) 12
H) 4x
I) undefined
11. Find \( f'(x) \) for \( f(x) = \frac{2x - 7}{3x + 1} \).

A) \( \frac{2}{3} \)

B) \( \frac{2}{3x + 1} \)

C) \( -\frac{23}{3(3x + 1)} \)

D) \( -\frac{5}{4} \)

E) \( \frac{12x - 19}{3x + 1}^2 \)

F) \(-19\)

G) \(7 - 2x + 2/(3x + 1)\)

H) \(-7\)

I) \(\frac{23}{(3x + 1)^2}\)
12. Find $f'(t)$ for $f(t) = 2t\sqrt{t^3 - 3t^2 + 5t}$.

A) $t(t^3 - 3t^2 + 5t)^{-1/2}$

B) $t(t^3 - 3t^2 + 5t)^{-1/2}(3t^2 - 6t + 5)$

C) $2t(t^3 - 3t^2 + 5t)^{-1/2}$

D) $2t(t^3 - 3t^2 + 5t)^{-1/2}(3t^2 - 6t + 5)$

E) $-t(t^3 - 3t^2 + 5t)^{-1/2}(3t^2 - 6t + 5)$

F) $t(t^3 - 3t^2 + 5t)^{3/2}(3t^2 - 6t + 5)$

G) $2\sqrt{t^3 - 3t^2 + 5t} - t(t^3 - 3t^2 + 5t)^{-1/2}(3t^2 - 6t + 5)$

H) $2\sqrt{t^3 - 3t^2 + 5t} + t(t^3 - 3t^2 + 5t)^{-1/2}(3t^2 - 6t + 5)$

I) $2\sqrt{t^3 - 3t^2 + 5t} - t(t^3 - 3t^2 + 5t)^{-1/2}$
13. Given $f(3) = 1$ and $f'(3) = -4$, find the derivative at $x = 3$ of $G(x) = (f(x))^9$.

A) 1
B) -4
C) $(-4)^8$
D) -36
E) $(-4)^9$
F) $9(f(x))^8$
G) $9(f(1))^8$
H) $9(f(x))^8f'(x)$
I) -16
14. Find the values of $x$ where the tangent line is horizontal for the function $f(x) = 6x - x^2$.

A) $6 - 2x$

B) 0.6

C) -6, 0

D) 6

E) -6

F) 0

G) 3

H) -3

I) tangent is never horizontal
Part II. True-False (1 point each) Mark your card “A” if the statement is true and “B” if the statement is false.

15. If \( y = f(x) \) is a polynomial, then you know \( y = f'(x) \) is a polynomial.

16. If \( f(-10) = -10 \) and \( f(10) = 10 \), then you know \( f(x) = 0 \) for some \( x \) in the interval \([-10, 10]\).

17. The function \( f(x) = \log_{10}(-x) \) is undefined at all \( x \).

18. The marginal cost \( C''(100) \) always gives a good estimate of the cost of producing one more item at a production level of 100 items.

19. If \( \lim_{x \to -5} g(x) = \infty \), then you know \( g(5) \) is undefined.

20. If \( y = f(x) \) is the equation of a rational function, then so is \( y = f'(x) \).
Part III: These are three "free response" problems worth a total of 24 points. Write your answers on the test pages. Show your work neatly and cross out irrelevant scratchwork, false starts, etc.

Please put your Washington University ID number on each of the following pages as they might be separated during grading. Do NOT put your name on these pages. Also, please add your Discussion Section Letter (available on your exam front cover sheet) on each page so that we can return papers through discussion sections.

WashU ID Number:__________Discussion Section Letter:__

21. Use the definition of derivative and the two-step limiting process to compute the derivative of \( f(x) = 4 - 2\sqrt{x} \). Show all work.
22. Suppose that in a given gourmet food store, people are willing to buy \( x \) pounds of chocolate candy per day at \( \$p \) per quarter pound, as given by the price-demand equation
\[
x = 10 + \frac{180}{p}, \quad 2 \leq p \leq 10.
\]
Find the demand and the instantaneous rate of change of demand with respect to price when the price is \$5. Write a brief verbal interpretation of these results.
23. For the function $y = f(x)$ given by the graph below, sketch the
graph of the function $y = f'(x)$. giving details showing how you arrived
at your sketch. Do not use your calculator for this problem.