Name:  

Please print above

Course: Math 127

Part of your name should be printed in large letters at the top of this page of your examination booklet. Your proctor can help you find your booklet if necessary. Make sure an answer card is on top of the booklet.

- make sure you have an adequate supply of PENCILS and ERASERS and your WASHINGTON UNIVERSITY photo ID card. Your proctors will check your ID card during the exam. Students without their ID will have their score withheld until they present their ID at the math office.

- PRINT your name and the course and exam number at the top of your card. Fill in your ID number in the appropriate boxes.

- Do not use any extra NOTES, BOOKS or SCRATCH PAPER. You should have ample space in your booklet for calculations. If you run out of space use the sides of the booklet pages for your work.

- CALCULATORS are only allowed if your instructor permits them.

- MARK your answer card neatly and make clean erasures. Sloppy cards will delay grading and result in your scores being withheld until you visit the math office to see your mismarkings.

- To see your exam score, go the the math department homepage at www.math.wustl.edu and use the link to 'Exam Scores' under 'Resources'.

- Scores on multiple choice questions will usually appear on the website within two days.

- Scores will also be posted by your course bulletin board in Cupples I.

For more information about your exam contact your instructor or the math department office in Cupples I, room 100.
MATH 127, FALL 2003
-EXAM 3-

Name: ____________________ WashU ID Number: ________

There are 18 questions worth a total of 100 points. The first 15 questions are worth 5 points each, for a total of 75 points. Questions 16 and 17 are worth 8 points each and question 18 is worth 9 points.

These are all “free response” problems. Write your answers on the test pages. Show your work neatly and cross out irrelevant scratchwork, false starts, etc.

Please put your name and WashU ID number in the spaces marked above. And please put your Washington University ID number on each of the following pages as they might be separated during grading. Do NOT put your name on these pages. Also, please add your Discussion Section Letter (available on your exam front cover sheet) on each page so that we can return papers through discussion sections.

No calculators with a CAS are allowed.
1. The demand equation for a certain item is
   \[ p = 10 - \frac{x}{2000} \]
   and the cost equation is
   \[ C(x) = 8000 + 3x. \]
   Find the marginal profit at a production level of 4000 and interpret the result.
2. The function $g(x)$ is continuous on $(-\infty, \infty)$ and has four critical values: $x = -2, -0.5, 3,$ and $10$. Use the given sign chart for $g'(x)$ to determine whether $g$ has a local minimum, a local maximum, or neither at each critical value.
3. Given $h(x) = x + \frac{16}{x}$, $x < 0$, find all values of $x$ corresponding to local extrema.
4. The cost of manufacturing $x$ blenders in one day is given by

$$C(x) = 2x^3 - 16x^2 + 4x.$$ 

Find the average cost per blender and the interval where the average cost per blender is decreasing.
5. Find $y''$ for $y = (x^2 - 16)^5$. 
6. Find all inflection points for $f(x) = x^4 - 10x^3 + 24x^2 + 3x - 1001$. 
7. The critical values for $f(x)$ are $x = -10$ and $x = 1$. Use the sign chart for $f''$ below to find all $x$-values (if any exist) for the local extrema of $f$. State whether each local extremum is a local maximum or a local minimum.

\[
\begin{array}{cccccc}
\text{Sign Chart for } f'' \\
\hline
- & - & - & 0 & + & + \\
-1 & & & & & \\
\end{array}
\]
8. Sketch a possible graph for $f$, where $f$ is a continuous function satisfying $f(0) = 3$ and has the sign chart information given below.
9. A company estimates that it will sell $N(t)$ hair dryers after spending $S \ t$ thousands on advertising as given by:

$$N(t) = -3t^3 + 450t^2 - 21,600t + 1,100 \quad 40 \leq t \leq 60.$$ 

For which values of $t$ is the rate of sales $N'(t)$ increasing? Express your answer using interval notation.
10. Find all horizontal asymptotes, if any exist, for
\[ f(x) = \frac{3x - 50,000x^2 + 1000001}{2x^2 + 30000x - 1} \].
11. Find all vertical asymptotes for
\[ f(x) = \frac{x - 5}{x^2 - 3x - 10}. \]
12. Sketch a graph of a function $f$ with all the following properties:
   - $f(0) = 8$, $f(2) = 8$
   - $f$ is continuous for all $x$ except $x = 1$
   - $x = 1$ is a vertical asymptote, $y = 4$ is a horizontal asymptote
   - $f'(x) > 0$ on $(-10, 1)$. $f'(x) < 0$ on $(1, 10)$
   - $f''(x) > 0$ on $(-10, 1) \cup (1, 10)$. 
13. Find all absolute extrema for \( f(x) = 5 + 4x + \frac{16}{x} \), \( x > 0 \).
14. Find $f'(x)$ for $f(x) = 2xe^x + 2\ln(x^3)$. 
15. Given \( f'(x) = 14 + \ln x \), for what values of \( x \) is \( f \) increasing? Express your answer using interval notation.
16. A large grocery chain found that on the average, a checker can recall \( P\% \) of a given price list \( x \) hours after starting work, as given approximately by

\[
P(x) = 96x - 24x^2. \quad 0 \leq x \leq 3.
\]

At what time does the checker recall a maximum percentage? What is this maximum?
17. Use the given graph of $y = f'(x)$ to sketch a graph of $y = f(x)$ such that $f(0) = 0$, giving details showing how you arrived at your sketch. Include all pertinent information. (Don’t forget to consider $f''$.)
18. Use the graphing strategy to sketch the graph of

\[ f(x) = \frac{x^2}{1 + 0.001x^2}. \]

Give all details and conclusions of the graphing strategy.