Math 128  Exam 1  Spring’10

You may use a (non-programable) scientific calculator for the exam, and you may use a 3 x 5 note card. This exam has 20 questions worth 5 points each. Indicate your answers on the answer card.

1. For \( f(x, y) = x^2 y \) and an unspecified number \( a \) find \( f(3, a + 1) \).
   (a) \( 9a \)
   (b) \( 9 \)
   (c) \( (a + 1)^2 \)
   (d) \( 9(a + 1)^2 \)
   (e) \( 6(a + 1) \)
   (f) cannot be determined from the given information
   (g) \( 9a^2 \)
   (h) \( 6a \)
   (i) \( 6a^2 \)
   (j) \( 9(a + 1) \)

2. Suppose \( F(x, y) = Ax^a y^{1-a} \) (\( A \) and \( a \) are unspecified constants). What is the relationship between \( F(3x, 3y) \) and \( F(x, y) \)?
   (a) \( F(3x, 3y) = F(x, y) \)
   (b) \( F(3x, 3y) = 3^a F(x, y) \)
   (c) \( F(3x, 3y) = 3^{-a} F(x, y) \)
   (d) \( F(3x, 3y) = a F(x, y) \)
   (e) \( F(3x, 3y) = 3a F(x, y) \)
   (f) \( F(3x, 3y) = F(x, y)/a \)
   (g) \( F(3x, 3y) = F(x, y)/3 \)
   (h) \( F(3x, 3y) = 3 F(x, y) \)
   (i) \( F(3x, 3y) = F(x, y)^3 \)
   (j) \( F(3x, 3y) = F(x, y)^a \)
3. Suppose

\[ f(x, y) = \frac{\sqrt{xy + 2}}{1 + x^3 y} \]

Which of the points \( A = (1, 1), B = (2, 2), C = (1, -3), D = (1, -1), E = (0, 0) \) are NOT in the domain of the function \( f \)?

(a) \( A, C \)
(b) \( C, D \)
(c) \( A, B \)
(d) \( B, D \)
(e) \( B, C \)
(f) \( A, D \)
(g) \( A, E \)
(h) \( B, E \)
(i) \( C, E \)
(j) \( D, E \)

4. For the function

\[ f(x, y) = x^3 + 3x^2 y^2 \]

What is

\[ \frac{\partial f}{\partial x} = f_x ? \]

(a) \( 3x^2 \)
(b) \( 3x^2 + 3x^2 y^2 \)
(c) \( 6xy^2 \)
(d) \( 6x^2 y \)
(e) \( 3x^2 + 6xy^2 \)
(f) \( x^3 + 6x^2 y \)
(g) \( x^3 + 6xy^2 \)
(h) \( 3x^2 + 12xy \)
(i) \( 6x + 6y^2 \)
(j) \( 6y^2 \)
5. For the same function $f(x, y) = x^3 + 3x^2y^2$

What is

$$\frac{\partial f}{\partial y} = f_y ?$$

(a) $3x^2$
(b) $3x^2 + 3x^2y^2$
(c) $6xy^2$
(d) $6x^2y$
(e) $3x^2 + 6xy^2$
(f) $x^3 + 6x^2y$
(g) $x^3 + 6xy^2$
(h) $3x^2 + 12xy$
(i) $6x + 6y^2$
(j) $6y^2x^2$

6. For the same function $f(x, y) = x^3 + 3x^2y^2$

What is

$$\frac{\partial \partial f}{\partial x \partial y} = f_{yx} ?$$

(a) $3x^2$
(b) $3x^2 + 3x^2y^2$
(c) $6xy^2$
(d) $6x^2y$
(e) $3x^2 + 6xy^2$
(f) $x^3 + 6x^2y$
(g) $x^3 + 6xy^2$
(h) $12xy$
(i) $6x + 6y^2$
(j) $6y^2x^2$
7. Suppose \[ f(x, y) = xy^2 + 1 \]

Which of the points \( A = (1, 1), B = (0, 1), C = (-1, 0), D = (-1, -1) \) are on the same level curve of \( f \) as the point \( (0, 0) \)?

(a) A, B
(b) A, C,
(c) A, D
(d) B, C
(e) B, D
(f) C, D
(g) A, B, C
(h) A, B, D
(i) A, C, D
(j) B, C, D

8. What is the distance between the points \((1, 2, 3)\) and \((2, 0, 2)\)?

(a) \( \sqrt{1} \)
(b) \( \sqrt{2} \)
(c) \( \sqrt{3} \)
(d) \( \sqrt{4} \)
(e) \( \sqrt{5} \)
(f) \( \sqrt{6} \)
(g) \( \sqrt{7} \)
(h) \( \sqrt{8} \)
(i) \( \sqrt{9} \)
(j) \( \sqrt{10} \)
9. What is the equation of the sphere of radius 3 and center \((2, 1, -1)\)?

(a) \(x^2 + y^2 + z^2 = 9\)
(b) \(x^2 + y^2 + z^2 = 4\)
(c) \(x + 2y + 3z = 9\)
(d) \(2x + y - z = 3\)
(e) \(2x + y - z = 9\)
(f) \((x - 2)^2 + (y - 1)^2 + (z + 1)^2 = 3\)
(g) \((x - 2)^2 + (y - 1)^2 + (z + 1)^2 = 9\)
(h) \(x^2y^2z^{-1} = 9\)
(i) \(2x^2 + y^2 - z^2 = 3\)
(j) \(2x^2 + y^2 - z^2 = 9\)

10. For the function \(g(x, y, z) = xe^{xy} + xe^{xz}\)

find

\[
\frac{\partial}{\partial y} g = g_y
\]

(a) \(yze^{xy} + e^{xz} + zxe^{xz}\)
(b) \(xze^{xy}\)
(c) \(2ze^{xy}\)
(d) \(2xe^{xz} + 2zxe^{xz}\)
(e) \(x^2yze^{xy}\)
(f) \(e^{xz} + zxe^{xz}\)
(g) \(yze^{xy} + zxe^{xz}\)
(h) \(yze^{xy} + e^{xz}\)
(i) \(yze^{xy} + e^{xz} + zxe^{xz}\)
(j) \(yze^{xy} + e^{xz} + yxe^{xz}\)
11. For the same function find

\( g(x, y, z) = xe^{xy} + xe^{xz} \)

\( \frac{\partial}{\partial x} g = g_x \)

(a) \( yxe^{xy} + e^{xz} + zxe^{xz} \)
(b) \( xze^{xy} \)
(c) \( 2zxe^{xz} \)
(d) \( 2xe^{xz} + 2zxe^{xz} \)
(e) \( x^2ye^{xy} \)
(f) \( e^{xz} + zxe^{xz} \)
(g) \( yze^{xy} + zxe^{xz} \)
(h) \( yze^{xy} + e^{xz} \)
(i) \( yze^{xy} + e^{xz} + zxe^{xz} \)
(j) \( yze^{xy} + e^{xz} + yxe^{xz} \)

12. For the same function find

\( g(x, y, z) = xe^{xy} + xe^{xz} \)

\( \frac{\partial^2}{\partial y^2} g = g_{yy} \)

\( \frac{\partial^2}{\partial y^2} g = \frac{\partial}{\partial y} \left( \frac{\partial}{\partial y} \left( xe^{xy} + xe^{xz} \right) \right) \)
\( = \frac{\partial}{\partial y} (zxe^y) \)
\( = zxe^y \)

(a) \( yze^{xy} + e^{xz} + zxe^{xz} \)
(b) \( xze^{xy} \)
(c) \( 2zxe^{xz} \)
(d) \( \Box x^2ze^{xy} \)
(e) \( e^{xz} + zxe^{xz} \)
(f) \( yze^{xy} + zxe^{xz} \)
(g) \( yze^{xy} + e^{xz} \)
(h) \( yze^{xy} + e^{xz} \)
(i) \( yze^{xy} + e^{xz} + zxe^{xz} \)
(j) \( yze^{xy} + e^{xz} + yxe^{xz} \)
13. Find the Hessian of the function \( f(x, y) = x^3 + 3xy \)

\[
\begin{array}{c}
a. \begin{pmatrix} 6 & 3 \\ 3 & 0 \end{pmatrix} \\
b. \begin{pmatrix} 6x & 3 \\ 3 & 0 \end{pmatrix} \\
c. \begin{pmatrix} 6x & 0 \\ 0 & 3xy \end{pmatrix} \\
d. \begin{pmatrix} 0 & 0 \\ 0 & 6x \end{pmatrix} \\
e. \begin{pmatrix} 3x^2 + 3y & 3x \\ 3x & 3x^2 + 3y \end{pmatrix} \\
f. \begin{pmatrix} x^3 & 0 \\ 0 & 3xy \end{pmatrix} \\
g. \begin{pmatrix} 1 & 3 \\ 3 & 0 \end{pmatrix} \\
h. \begin{pmatrix} 3 & x \\ y & xy \end{pmatrix} \\
i. \begin{pmatrix} 3xy & 0 \\ 0 & 6x \end{pmatrix}
\end{array}
\]

\[
\begin{pmatrix}
\ell_{xx} & \ell_{xy} \\
\ell_{yx} & \ell_{yy}
\end{pmatrix} = \begin{pmatrix} 6x & 3 \\ 3 & 0 \end{pmatrix}
\]

14. Suppose \( Y = F(K, L, T) \) is an agricultural production function where \( Y \) is the number of units of output produced, \( K \) is the capital investment, \( L \) is the labor input and \( T \) is the area of land used. Suppose that \( \partial Y / \partial K = 5 \). Which of the following are true?

i. The marginal product of capital is 5
ii. The marginal product of capital is positive
iii. An increase of \( K \) of 2 units would lead to an increase in output of approximately 10 units.
iv. A decrease in \( K \) would be expected to lead to a decrease in the output.

\( \square \) (a) All
(b) None
(c) ii and iv
(d) i and ii
(e) iii and iv
(f) i and iii
(g) i, ii, and iii
(h) i, ii, and iv
(i) i, iii, and iv
(j) ii, iii, and iv
15. \( A, B \) and \( a \) are positive constants; \( Y = AK^a + BL^a \). Compute and simplify the expression \( K \frac{\partial Y}{\partial K} + L \frac{\partial Y}{\partial L} \).

The result is?

(a) 0

(b) \( a \)

(c) \( Y \)

(d) \( aY \)

(e) \( A + B \)

(f) \( AB \)

(g) \( A^aB^a \)

(h) \( (KL)^a \)

(i) \( a(KL) \)

(j) \( a(K + L) \)

\[ = K (A aK^{a-1}) + L (B aL^{a-1}) \]

\[ = A aK^{a} + B aL^{a} \]

\[ = a \left( A K^a + B L^a \right) \]

\[ = aY \]

16. Suppose \( z = e^{xy^2} \). Compute \( E_{x} z \), the partial elasticity of \( z \) with respect on \( x \)

(a) \( x \)

(b) \( y \)

(c) \( y^2 \)

(d) \( xy^2 \)

(e) \( \log x \)

(f) \( \log y \)

(g) \( \log x + \log y \)

(h) \( \log x + 2 \log y \)

(i) \( e^x \)

(j) \( e^{xy^2} \)

\[ E_{x} z = \frac{x \frac{\partial z}{\partial x}}{z} \]

\[ = \frac{x}{e^{xy^2}} \cdot y^2 e^{xy^2} \]

\[ = xy^2 \]
17. At a certain time the demand for potatoes was estimated to be

\[ D = Ap^{-3}m^4 \]

where \( p \) is the price of potatoes and \( m \) is mean income. Find the price elasticity of demand, \( \text{El}_p D \).

(a) .4
(b) -.4
(c) .7
(d) -.7
(e) -.12
(f) .12
(g) .1
(h) -.1
(i) .3
(j) -.3

18. The annual herring catch is given by the function \( Y(K, S) = .06K^{1.3}S^{-6} \). Where \( K \) is the catching effort and \( S \) is the size of the herring stock. What will happen to the catch if both the effort and stock increase by a factor of three?

(a) The catch will not change.
(b) The catch will increase by a factor of 3
(c) The catch will increase by a factor of 3^{1.3}
(d) The catch will increase by a factor of 3^{6}
(e) The catch will increase by a factor of 3^{1.9}
(f) The catch will decrease by a factor of 3
(g) The catch will decrease by a factor of 3^{1.3}
(h) The catch will decrease by a factor of 3^{6}
(i) The catch will decrease by a factor of 3^{1.9}
(j) Cannot be determined from the given information.
19. The set of points \((x, y, z)\) in three dimensional space whose coordinates satisfy the equation \(x + y + z = 3\) form which of the following surfaces?

(a) A sphere centered at the origin, radius \(\sqrt{3}\).
(b) A sphere centered at \((1, 1, 1)\) radius \(\sqrt{3}\)
(c) A plane through the origin
(d) A plane through the point \((1, 1, 1)\)
(e) A curved surface through the origin
(f) A curved surface through the point \((1, 2, -1)\)
(g) A sphere centered at the origin radius 3
(h) A sphere centered at the point \((1, 1, 1)\) with radius 3

1) \((1, 1, 1)\) is on the surface
2) \((1, 2, -1)\) is not
3) Origin \((0, 0, 0)\) is not

1) eliminates b, 9, y
2) eliminates f
3) eliminates c, e

so answer is a) or d). Equation is not e.g. of a sphere so
ans = d

20. The straight line with equation \(x - y = 1\) is part of a single level curve for which of the following surfaces?

i. \(z = x + y\)
ii. \(z = x - y\)
iii. \(z = x^2 + y^2\)
iv. \(z = 3e^{x-y}\)

(a) All
(b) None
(c) i
(d) ii
(e) iii
(f) iv
(g) i and ii
(h) ii and iv
(i) i and iii
(j) iii and iv

\[\text{if}\ x-y \geq 1\]

\[\text{for i} \quad z = x+y \quad \text{could be anything}\]
\[z = x-1 = 1 = \text{constant}\]
\[\text{for iii} \quad z = x^2 + y^2 \quad \text{could be lots of things}\]
\[z = 3e^{x-y} = 3e^1 = \text{constant}\]

\[\text{Ans} \ ii + iv\]