Part I consists of 14 multiple choice questions (worth 5 points each) and 5 true/false question (worth 1 point each), for a total of 75 points. Mark the correct answer on the answer card. For Part I, only the answer on the card will be graded.

1. If \( y = \frac{x^3}{64} + \sqrt[3]{x} \), what is \( \frac{dy}{dx} \big|_{x=8} \) ?

A) \( \frac{4}{3} \)  
B) \( \frac{37}{12} \)  
C) \( \frac{40}{8} \)  
D) \( \frac{69}{64} \)  
E) \( \frac{75}{12} \)  
F) \( \frac{3}{8} \)  
G) \( \frac{7}{12} \)  
H) \( \frac{35}{8} \)  
I) \( \frac{127}{64} \)  
J) \( \frac{14}{3} \)

2. The slope of the tangent line to \( f(t) = \frac{a + e^t}{e^t} \) where \( t = 0 \) is 5. What is \( a \) ?

A) 0  
B) \(-1\)  
C) \(-2\)  
D) \(-3\)  
E) \(-4\)  
F) 5  
G) 4  
H) 3  
I) 2  
J) 1
3. Suppose \( F(x) = f(g(x)) \) and that:

\[
\begin{align*}
g(2) &= 6 \\
g'(2) &= 3 \\
f'(2) &= -1 \\
f(2) &= 7 \\
f'(6) &= -4
\end{align*}
\]

what is \( F'(2) \)?

A) 0  B) 2  C) 21  D) 42  E) -24
F) 4  G) 18  H) -6  I) -3  J) -12

4. If \( g(4) = 2 \) and \( g'(4) = 3 \), what is \( \left( \frac{\sqrt{x}}{g(x)} \right)'(4) \)?

A) \( \frac{1}{2} \)  B) \( -\frac{4}{3} \)  C) \( \frac{14}{9} \)  D) \( -\frac{13}{4} \)  E) \( -\frac{11}{5} \)
F) \( \frac{33}{2} \)  G) -4  H) \( \frac{17}{6} \)  I) \( -\frac{17}{3} \)  J) 3

5. The figure shows the graph of two functions \( f \) and \( g \).
Let \( h(x) = f(x)g(x) \). What is \( h'(2) \)?

A) \( \frac{1}{2} \)  B) \( -\frac{4}{3} \)  C) \( \frac{14}{5} \)  D) \( -\frac{13}{4} \)  E) \( -\frac{11}{6} \)  
F) \( \frac{23}{2} \)  G) \( -4 \)  H) \( \frac{17}{6} \)  I) \( -\frac{17}{3} \)  J) 3

6. At time \( t \) (hrs), the size \( P \) of a certain population of bacteria is \( P = 5^{t^2} + t \). How fast is \( P \) changing at time \( t = 1 \)? (Round your answer to the nearest integer. All answers have the units "bacteria/hr").

A) 23  B) 130  C) 25  D) 18  E) 137  
F) 143  G) 121  H) 97  I) 243  J) 87

7. A point is moving along a straight line. At time \( t \) its velocity \( v(t) = t^2 - 3t + 2 \).
Exactly two of the following statements are true. Which ones are true?

i) The point is moving in the positive direction for times \( t < 1 \).
ii) The graph of the position function \( s = f(t) \) is always concave up.
iii) During the times \( 8 < t < 10 \), the point is speeding up.
iv) When \( \frac{ds}{dt} \) is increasing, the point must be moving in the positive direction.
v) With the information given, it is possible to calculate the position at time \( t = 0 \)

A) i, ii  
B) i, iii  
C) i, iv  
D) i, v  
E) ii, iii

F) ii, i  
G) ii, v  
H) iii, iv  
I) iii, v  
J) iv, v

8. If \( g(x) = \sec^2(2x) \), what is \( g'(\frac{\pi}{6}) \)?

A) 0  
B) 1  
C) 2  
D) \( \frac{1}{2} \)  
E) \( \sqrt{2} \)

F) \( \frac{1}{\sqrt{2}} \)  
G) 8  
H) \( \frac{1}{8} \)  
I) \( \frac{\sqrt{2}}{2} \)  
J) \( \frac{2}{\sqrt{3}} \)

9. Suppose \( f(3) = 1 \) and \( f'(3) = 2 \). What is the estimated value, using linear approximation, for \( f(2.99) \)?
10. The cost ($\text{)}) of producing $x$ toasters at the G.W. Crumbly Factory is $C(x) = 1000000 - 0.0001(x - 90000)^2$. What is the marginal cost when $x = 30000$? (Round your answer to the nearest cent.)

A) 12.87  B) 12.97  C) 14.32  D) 13.00  E) 15.67  
F) 13.69  G) 18.67  H) 11.43  I) 12.00  J) 10.45
11. The figure below shows the graph of the derivative \( y = f'(x) \) for some function \( y = f(x) \).

Exactly two of the following statements are true. Which ones are true?

i) \( f(x) \) has an inflection point at \( x = 3 \)
ii) \( f(x) \) has a local maximum at \( x = 4 \)
iii) \( f''(x) \) is increasing for \( 5 < x < 6 \)
iv) \( f(x) \) is concave down for \( 3 < x < 5 \)
v) \( f''(x) < 0 \) for \( 1 < x < 3 \)

A) i, ii  B) i, iii  C) i, iv  D) i, v  E) ii, iii
F) ii, iv  G) ii, v  H) iii, iv  I) iii, v  J) iv, v
12. Let \( y = xe^{-8x^2} \). What is the smallest value of \( a \) that makes the statement "\( y \) is decreasing for \( x > a \)" true?

A) 8  B) 4  C) 2  D) \( \frac{1}{2} \)  E) \( \frac{1}{4} \)

F) 0  G) \( -\frac{1}{4} \)  H) \( -\frac{1}{2} \)  I) \( -1 \)  J) \( -2 \)

13. What is \( \lim_{h \to 0} \frac{(1+h)^{10000} - 1}{h} \)?

A) 0  B) 10000  C) \( e \)  D) 100  E) 1

F) \( \frac{1}{e} \)  G) \( \frac{1}{100} \)  H) \( \frac{1}{10000} \)  I) \( 2^{10000} \)  J) \( \infty \) (limit d.n.e.)
14. There is one (and only one) line through the point (2, 1) that is tangent to the graph of \( y = \frac{x}{x^2 - 1} \) at some point \( P \) on the graph. What is the \( x \)-coordinate of \( P \)?

A) \( \frac{3}{2} \)  B) 1  C) 2  D) -1  E) -2
F) \( \frac{1}{4} \)  G) \( -\frac{2}{3} \)  H) \( \frac{5}{3} \)  I) 3  J) \( \frac{3}{4} \)

Questions 15-19 are true/false questions.

15. If \( \lim_{h \to 0} f(2 + h) = f(2) \), then \( f \) must have a derivative at 2.

A) True  B) False

16. There is one and only one point where the function \( f(x) = |x^2 + 1| + |x - 2| \) fails to have a derivative.

A) True  B) False
17. Suppose $T, P, V$ (the temperature, pressure, and volume of a gas in a container) are related by $\frac{PV}{T^2} = k$, where $k$ is a constant. If the temperature is held constant, then $\frac{dV}{dP} = -\frac{kT}{P^2}$.

A) True  B) False

18. Suppose $f'(x) = (x - 2)^2(x - 5)$. Then $f(x)$ has either a local maximum or a local minimum at $x = 2$.

A) True  B) False

19. If $f$ has a derivative at 3, then $\lim_{x \to 3} f(x) = f(3)$.

A) True  B) False
20. For each function \( y = f(x) \) given below, find the derivative. After all the differentiation is completed, no further simplifications are necessary. For example, an answer that looking like

\[
\frac{dy}{dx} = \frac{2(3\cos(2x) - 4x(2x + 1))(3)}{(x + 1)(x + 3) - (2)(3)x}
\]

would require no further simplification.

a) \( f(x) = \frac{(3x - 4)^{10}}{\cos x} \)

b) \( f(x) = 5^{\sec(\sqrt{x})} \)

c) \( f(x) = \tan(\sin(e^{3x^2})) \)
21. The graph of a function \( y = f(x) \) is shown below. On the grid beneath it, draw a reasonable graph for \( f'(x) \). Be sure your picture clearly indicates the value of \( f'(2) \) (the tangent line at \( x = 2 \) is drawn to help you), the places where the derivative is 0, the places where the derivative does not exist, and where \( f'(x) \) is increasing or decreasing. If those things are done, then the precise shape of \( f'(x) \) is not important. If you are estimating slopes, be sure to look at the scale on each axis.