Math 131 **Exam 4 (Final Exam)** **F10M131.4.1**

Name ______________________________ **ID Number** ____________________

You may have a simple scientific calculator to assist with arithmetic, but no graphing calculators are allowed on this exam.

The exam consists of 18 multiple choice questions (worth 5 points each) and 10 true/false questions (worth 1 point each). Mark your answers for all the questions on the answer card supplied. Only the answers on the card will be graded.

1. Find the right endpoint Riemann sum, R_4, that approximates $\int_2^{10} (10 - x)\, dx$ and tell whether it overestimates or underestimates the value of the integral.

A) 12, overestimate B) 12, underestimate C) 16, overestimate D) 16, underestimate

E) 20, overestimate F) 20, underestimate G) 22, overestimate H) 22, underestimate

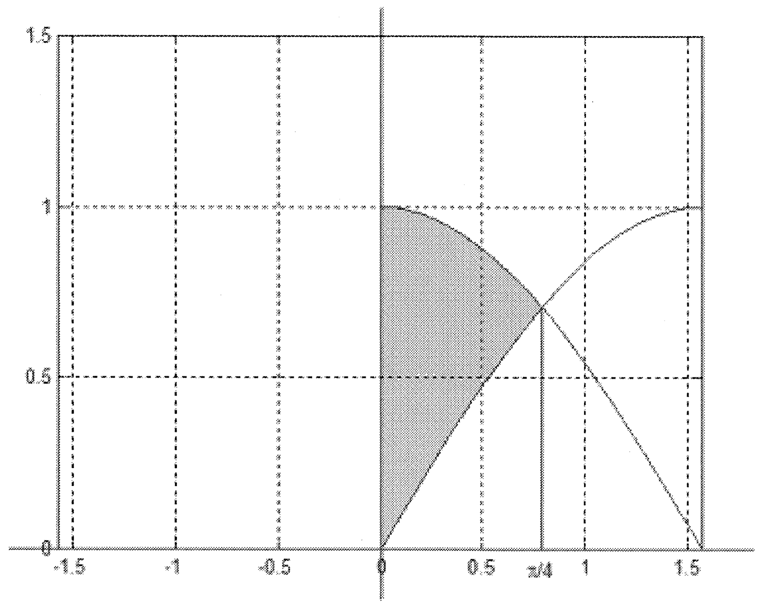
I) 24, overestimate J) 24, underestimate

2. If $y = f(x) = \frac{xe^x}{1+x}$, what is $f'(1)$?

A) 1 B) e C) 0 D) $2e$ E) $\frac{1}{e}$

F) $\frac{3e}{4}$ G) $\frac{e^2}{2}$ H) $\frac{2e+1}{4}$ I) $\frac{e}{1+e}$ J) $\frac{e}{(1+e)^2}$

3. Find the shaded area between the graphs of $y = \cos x$ and $y = \sin x$ above the interval $[0, \frac{\pi}{4}]$.



A) 2 B) $\sqrt{2} - 1$ C) $\sqrt{2} + \frac{1}{2}$

D) $\sqrt{3} - \sqrt{2}$ E) $\frac{1}{16}$ F) $\sqrt{2} - \frac{\sqrt{3}}{2}$

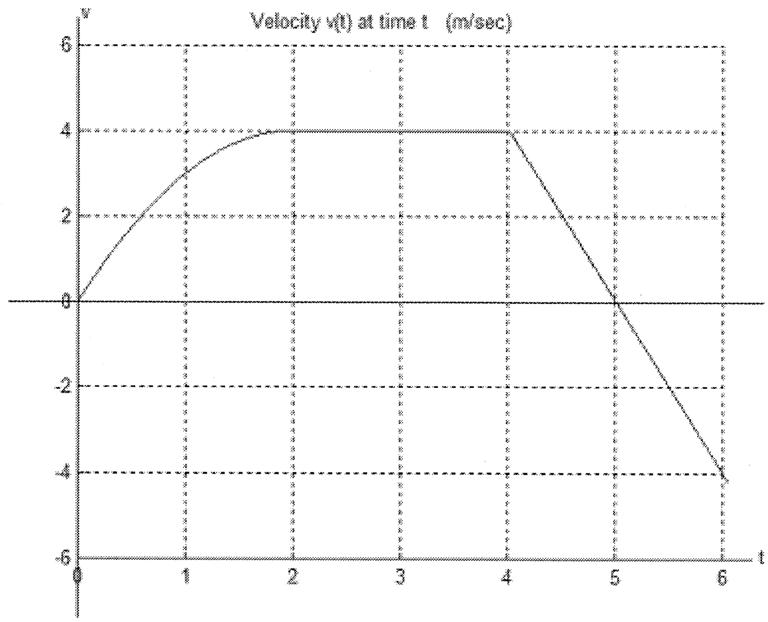
G) $\frac{1}{8}$ H) $\frac{1}{2}$ I) $\frac{\sqrt{2}+1}{2}$ J) $\frac{\sqrt{2}}{2} - \frac{1}{\sqrt{2}}$

4. Using a linear approximation (or equivalently, a differential) estimate the value of $\sqrt{36.2}$.
(*Round your answer to 4 decimal places.*)

A) 5.9832 B) 5.9892 C) 5.9979 D) 6.0002 E) 6.0167

F) 6.0322 G) 6.0411 H) 6.0497 I) 6.0503 J) 6.0531

5. A particle moves along a straight line. For $0 \leq t \leq 2$, its velocity is $v(t) = 4 - (t-2)^2$, and for $4 \leq t \leq 6$, the graph of $v(t)$ consists of straight line segments, as shown in the figure below. The velocity is measured in m/sec.



What is the net displacement of the particle during the time interval $0 \leq t \leq 6$?
(*Caution: notice that the "scale marks" along the x-axis are at* $0, 1, 2, 3, \ldots$
but are at $0, 2, 4, 6$ *along the y-axis.*)

A) $\frac{18}{3}$ m B) $\frac{40}{3}$ m C) $\frac{52}{3}$ m D) $\frac{46}{3}$ m E) $\frac{37}{3}$ m
F) $\frac{13}{3}$ m G) 0 m H) $-\frac{22}{3}$ m I) $-\frac{46}{3}$ m J) $-\frac{28}{3}$ m

6. Suppose $\int_0^3 2f(t)\,dt = 4$, $\int_1^3 3f(t)\,dt = -3$, and $\int_{-1}^1 4f(t)\,dt = 12$.

What is $\int_{-1}^0 f(t)\,dt$?

A) 0 B) -3 C) -3 D) -2 E) -1
F) 1 G) 2 H) 3 I) 4 J) 5

7. What is the value of $\int_0^1 \frac{x+4}{x^2+8x+1}\,dx$?

A) $\ln 3$ B) $\frac{1}{2}\ln 3$ C) $\ln \frac{3}{2}$ D) $2-\ln 3$ E) $3-\ln 10$

F) $\ln 8$ G) $\frac{1}{2}\ln 8$ H) $\frac{\ln 3}{\ln 2}$ I) $\frac{\ln 10}{2}$ J) $\ln\left(\frac{5}{10}\right)$

8. The mass (g) of a certain radioactive isotope is disappearing due to radioactive decay. If A is the mass present at time t (years), then $\frac{dA}{dt} = -70e^{-0.7t}$ grams/year.

How much of the isotope disappears during the time interval $0 \le t \le 2$? (*Round your answer to the nearest tenth.*)

A) 25.2 g B) 54.2 g C) 75.3 g D) 84.6 g E) 95.3 g

F) 101.2 g G) 108.9 g H) 111.3 g I) 117.8 g J) 123.1 g

9. The function $f(x) = x^2e^{-2x}$ has exactly two critical points, at $x = a$ and $x = b$. Let a be the smaller of these two critical points. Which one of the following is true:

A) f has a local maximum at a and a local minimum at b

B) f has a local maximum at a but neither a local maximum nor minimum at b

C) f has a local maximum at both a and b.

D) f has a local minimum at a and a local maximum at b

E) f has a local minimum at a but neither a local maximum nor minimum at b

F) f has a local minimum at both a and b

G) f has no local maximum or minimum values

H) f has a local minimum at a and an inflection point at b

I) f has an inflection point at a and a local maximum at b

J) f has inflection points at both a and b.

10. Suppose a, b are constants. What is $\lim_{x \to 0} \frac{\cos ax - e^{bx}}{bx^2 + ax}$?

A) $a - b$ B) $a + b$ C) ab D) $\frac{e^b}{2b}$ E) $\frac{e^b}{a}$

F) $\frac{a}{b}$ G) $-\frac{b}{a}$ H) $\frac{b}{b+a}$ I) $\frac{a-b}{a+b}$ J) $\frac{be^b}{a}$

11. What is the <u>smallest</u> possible value for a that makes the following statement true?

$$F(x) = \int_3^{x^2+6x} (t^2+1)(t^2+4)\,dt \text{ is } \underline{\text{increasing}} \text{ on } (a, \infty)$$

A) $a = -4$ B) $a = -3$ C) $a = -2$ D) $a = -1$ E) $a = 0$
F) $a = 1$ G) $a = 2$ H) $a = 3$ I) $a = 4$ J) $a = 5$

12. Using the definition of derivative, which of the following is given by

$$\lim_{h \to 0} \frac{\sin(\pi(2+h))}{h} \quad ?$$

A) $f'(0)$ where $f(x) = \sin(2\pi x)$ B) $f'(2)$ where $f(x) = \sin(\pi x)$

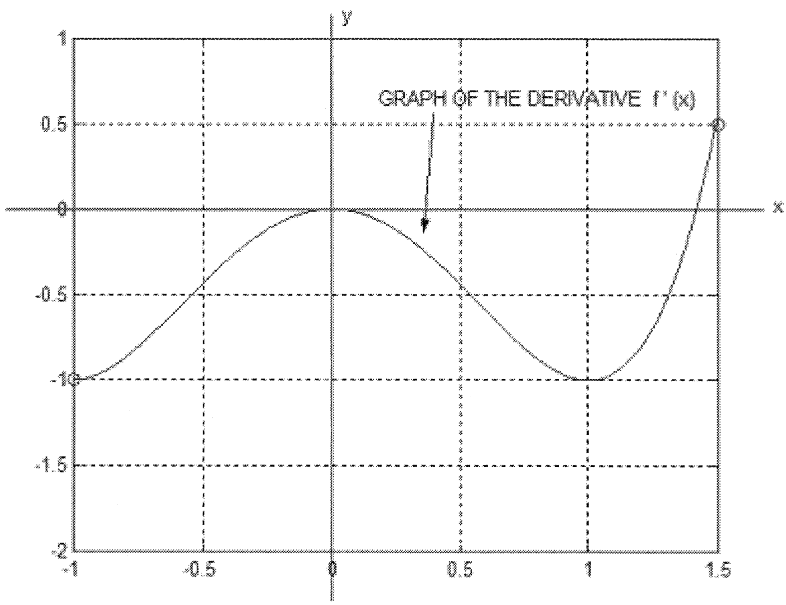
C) $f'(2\pi)$ where $f(x) = \sin(x)$ D) $f'(\pi)$ where $f(x) = \sin(2\pi x)$

E) $f'(\pi)$ where $f(x) = \sin(\pi x)$ F) $f'(0)$ where $f(x) = \sin(2x)$

G) $f'(\frac{\pi}{2})$ where $f(x) = \sin(x)$ H) $f'(\frac{\pi}{2})$ where $f(x) = \sin(2\pi x)$

I) $f'(2\pi)$, where $f(x) = \frac{\sin(x)}{x}$ J) $f'(2\pi)$ where $f(x) = \frac{\sin(2x)}{x}$

13. The graph below shows the <u>derivative</u> $f'(x)$ of some function $y = f(x)$.
$f(x)$ is defined on the interval $(-1, 1.5)$.



Which of the following statements are true?

i) $f(x)$ has exactly one local maximum
ii) $f(x)$ has a local minimum at $x = 0$
iii) $f(x)$ has exactly two inflection points
iv) $f(-0.5) > f(1)$
v) f has an absolute minimum value that occurs at some point $x = c$, where $c < 0$.

A) only i), ii) are true

B) only i), iii) are true

C) only i), iv) are true

D) only i), v) are true

E) only ii), iii) are true

F) only ii), iv) are true

G) only ii), v) are true

H) only iii), iv) are true

I) only iii), v) are true

J) only v) is true.

14. If $y = f(x) = \ln\left(\frac{2\sin x}{3+4\cos x}\right)$, what is $f'(\frac{\pi}{2})$?

A) $\frac{1}{2}$ B) $\frac{2}{3}$ C) $\frac{2}{7}$ D) $\frac{4}{3}$ E) 2

F) $\frac{7}{2}$ G) 1 H) 0 I) $\frac{1}{6}$ J) $\frac{1}{3}$

15. We want to construct a box with a rectangular base. The box includes a top (*that is, it's not "open" on the top.*) The volume must be 50 m^3 and the length of the base must be three times its width.

The material used to build the top and bottom cost \$10/m^2 and the material used to build the sides cost \$6/m^2. How should we choose the width of the base to minimize the cost of building the box? (*Round your answer to two decimal places.*)

A) $w = 0.55$ m B) $w = 0.78$ m C) $w = 0.92$ m D) $w = 1.14$ m
E) $w = 1.47$ m F) $w = 1.53$ m G) $w = 1.62$ m H) $w = 1.73$ m
I) $w = 1.79$ m J) $w = 1.88$ m

16. Let $s = f(t) = \frac{18}{1+2t^2}$ be the position (m) at time t (sec) of a point moving along a straight line. According to the Mean Value Theorem, there must be a time $t = c$ between $t = 1$ and $t = 2$ when the point has velocity $v(c) = ???$.

A) 4 m/sec B) 3.5 m/sec C) 3 m/sec D) 2 m/sec

E) 0 m/sec F) − 2.5m/sec G) − 4 m/sec H) − 4.5 m/sec

I) − 5 m/sec J) − 5.5 m/sec

17. What is the value of $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{1}{\sqrt{1+\frac{2i}{n}}} \cdot \frac{2}{n} \right)$?

A) 0 B) 2 C) $\sqrt{2}-1$ D) $2(\sqrt{5}-1)$ E) 4

F) $2(\sqrt{3}-1)$ G) $\frac{1}{2\sqrt{3}}$ H) $\frac{1}{\sqrt{2}+1}$ I) 5 J) $\sqrt{6}$

18. Suppose g is a differentiable function for which

$$g(0) = 0 \qquad g(2) = \tfrac{\pi}{4}$$
$$g'(0) = 2 \qquad g'(2) = 2\sqrt{2}$$

Let $H(x) = \left(\sin(g(x))\right)^3 + e^{g(x-2)}$. What is $H'(2)$?

A) 5 B) 4 C) 3 D) 2 E) 1

F) 0 G) -1 H) -2 I) -3 J) -4

True/False:

19. Suppose $y = f(x)$ is defined on $[13, 17]$ and that $\frac{dy}{dx} = \frac{x^{18} + 23}{e^x}$. Then f must have both an absolute maximum and an absolute minimum value, and the absolute maximum value must occur at the right endpoint, $x = 17$.

A) True B) False

20. If f is continuous at $x = a$, then f must be differentiable at $x = a$.

A) True B) False

21. $\frac{d}{dx}\tan^2 x = \frac{d}{dx}\sec^2 x$

A) True B) False

22. If $\lim_{x\to\infty} f(x) = \infty$ and $\lim_{x\to\infty} g(x) = \infty$, then $\lim_{x\to\infty}(f(x) - g(x)) = 0$

A) True B) False

23. If $G(x) = x^{29}$, then $\lim_{h\to 0}\frac{G(1+h) - G(1)}{h} = 29$

A) True B) False

24. If $f'(r)$ exists, then $\lim_{x\to r} f(x) = f(r)$

A) True B) False

25. If $f(x)$ and $g(x)$ are decreasing differentiable functions, then the function $H(x) = f(g(x))$ is increasing.

A) True B) False

26. If $f''(2) = 0$, then the point $(2, f(2))$ must be an inflection point on the graph of $y = f(x)$.

A) True B) False

27. There is a function $f(x)$ for which $f(1) = -2$, $f(3) = 0$ and $f'(x) > 1$ for every x.

A) True B) False

28. If $f(x)$ is continuous on $[a, b]$, then $\frac{d}{dx}\left(\int_a^b f(x)dx\right) = f(x)$.

A) True B) False