Math 131
Final Examination – May 1, 2009

Name ___________________________

General Instructions: You may use a simple calculator that is not graphing or programmable. You may have up to four 3x5 cards, but no other notes.

Part I (60 points): For each of the following 17 problems, mark your answer on the answer card. For Part I, only the answer on the card will be graded.

Problems 1-10: Multiple choice. Each problem is worth 5 points.

1. Evaluate $\int_0^{\pi/6} \sin 3x \, dx$

   (a) 2
   (b) 3/2
   (c) 4/3
   (d) 1
   (e) 1/2
   (f) 1/3
   (g) 0
   (h) ∞

2. Find the equation of the tangent line to $f(x) = \ln 2x + x^2 - \ln 2$ at the point $(1, 1)$.

   (a) $y = 1$
   (b) $y = 2.5(x - 2) + 1 = 2.5x - 4$
   (c) $y = 2.5(x - 1) + 2 = 2.5x - 0.5$
   (d) $y = 2.5(x - 1) + 1 = 2.5x - 1.5$
   (e) $y = 3(x - 2) + 1 = 3x - 5$
   (f) $y = 3(x - 1) + 2 = 3x - 1$
   (g) $y = 3(x - 1) + 1 = 3x - 2$
   (h) $x = 1$

3. Find all points where the tangent line to the graph of $\dfrac{x^3}{x^2 - 3}$ is horizontal.

   (a) 0, $\sqrt{3}$ and $-\sqrt{3}$
   (b) 0 and $\sqrt{3}$
   (c) 0, 3, and $\sqrt{3}$
   (d) 0, 3, -3, $\sqrt{3}$, $-\sqrt{3}$
   (e) 0, 3, and -3
   (f) 0 and 3
   (g) 0, π, $-\pi$
   (h) No such points.

4. Calculate $\lim_{x \to 0} \frac{x^2}{\sin(x^2 - x)}$

(a) 0
(b) 1
(c) -1
(d) $1/2$
(e) π
(f) $-\infty$
(g) ∞
(h) undefined

5. Calculate $\lim_{x \to 0} \frac{x^2}{e^x}$

(a) 0
(b) 1
(c) -1
(d) e^2
(e) e
(f) $-\infty$
(g) ∞
(h) undefined

6. Find the maximum value of $f(x) = e^x + e^{-x}$ on the interval $[-3, 3]$.

(a) $e^3 - e^{-3}$

(b) $2\ln 3$

(c) 2

(d) 21

(e) $e + e^{-1}$

(f) 30

(g) $e^3 + e^{-3}$

(h) ∞

7. Which of the following is the Riemann sum with a uniform partition and right endpoints representing $\int_1^4 \ln x\, dx$?

(a) $\sum_{k=1}^{n} \ln(1 + \frac{k}{n}) \cdot \frac{1}{n}$

(b) $\sum_{k=1}^{n} \ln(1 + \frac{4k}{n}) \cdot \frac{4}{n}$

(c) $\sum_{k=1}^{n} \ln(\frac{4k}{n}) \cdot \frac{4}{n}$

(d) $\sum_{k=1}^{n} \ln(1 + \frac{k-1}{n}) \cdot \frac{1}{n}$

(e) $\sum_{k=1}^{n} \ln(\frac{k}{n}) \cdot \frac{1}{n}$

(f) $\sum_{k=1}^{n} \ln(\frac{4(k-1)}{n}) \cdot \frac{4}{n}$

(g) $\sum_{k=1}^{n} \ln(1 + \frac{3k}{n}) \cdot \frac{3}{n}$

(h) $\ln 4 - \ln 1$

8. Which of the following is the ϵ-δ definition of the statement

$$\lim_{x \to 0} e^x = 1?$$

(a) for all $\epsilon > 0$ there exists a $\delta > 0$ such that $0 < |x-1| < \delta \implies |e^x| < \epsilon$.

(b) for all $\epsilon > 0$ there exists a $\delta > 0$ such that $0 < x-1 < \delta \implies |e^x| < \epsilon$.

(c) for all $\epsilon > 0$ there exists a $\delta > 0$ such that $0 < |x| < \delta \implies |e^x - 1| < \epsilon$.

(d) for all $\epsilon > 0$ there exists a $\delta > 0$ such that $0 < x < \delta \implies |e^x - 1| < \epsilon$.

(e) for all $\epsilon > 0$ there exists a $\delta > 0$ such that blah blah blah math greek blah blah.

(f) for all $\epsilon < 0$ there exists a $\delta < 0$ such that $0 < |x-a| < \delta \implies |f(x) - L| < \epsilon$.

(g) Undefined/doesn't exist.

(h) Rabbit.

9. Let $f(x) = e^{x^2}$, and $F(x)$ be any antiderivative of f. Find all critical points of F.

(a) e, 0, and e^{-1} only.

(b) 0 and e^{-1} only.

(c) e and 0 only

(d) e and e^{-1} only.

(e) e^{-1} only.

(f) e only.

(g) 0 only

(h) F has no critical points.

10. At what points does $f(x) = \dfrac{(x-2)\sin x}{x^2 - x}$ have a vertical asymptote?

(a) At 1, 0, and -1 only.
(b) At 0 and -1 only.
(c) At 1 and 0 only
(d) At 1 and -1 only.
(e) At -1 only.
(f) At 1 only.
(g) At 0 only
(h) $f(x)$ has no vertical asymptotes.

Problems 11-15: True/false. Each problem is worth 2 points.

11. True/false: if f is an increasing function on $(-\infty, 1)$ and a decreasing function on $(1, \infty)$, then the derivative of f is defined at 1 and $f'(1) = 0$.

(a) True
(b) False

12. True/false: If f is continuous at 3, then the limit $\lim_{x \to 3+} f(x)$ exists.

(a) True
(b) False

13. True/false: a Riemann sum $\sum_{k=1}^{n} f(c_k)\Delta x_k$ for f on $[a, b]$ is an antiderivative for f on the same interval.

(a) True
(b) False

14. True/false: The definite integral $\int_0^3 (2 - e^x)\, dx$ represents an area.

   (a) True
   (b) False

15. True/false: if f is differentiable everywhere, then it has an antiderivative.

   (a) True
   (b) False

Part II (40 points): In each of the following problems, show your work clearly in the space provided. Partial credit will be given, and a correct answer without supporting work may not receive credit.

1. (3 points) In 2-4 sentences, explain why limits are an important concept in Math 131.

2. Consider the function $y(t) = e^t + e^{-t}$.

(a) (4 points) Where is $y(t)$ rising? Where is it falling?

(b) (4 points) Where is $y(t)$ concave upward? Concave downward?

(c) (3 points) What is $\lim_{t \to \infty} y(t)$? What is $\lim_{t \to -\infty} y(t)$?

(d) (3 points) Graph the above function $y(t)$. Identify on your graph all critical points and inflection points, as well as the limits from part (c).

3. A circus performer throws a knife into the air. Gravity provides a constant acceleration of $-4.9\ \mathrm{m}/\mathrm{s}^2$. The initial velocity at $t = 0$ is 6 m / s, and height at $t = 0$ is 1 m.

   (a) (6 points) Find the vertical velocity $v(t)$ and height $s(t)$ of the knife at time t.

   (b) (3 points) Find all critical points of $s(t)$. When is the knife at its highest point?

   (c) (2 points) At what time does the knife hit the ground? (height 0)

4. (12 points) Calculate the following, showing all your work:

(a) $\displaystyle\int \sin x \cdot \cos^2 x \, dx$

(b) $\displaystyle\int \frac{e^{2x}}{1 - e^x} \, dx$