This exam should have 22 questions. There are 2 parts. The first part is the 20 multiple choice problems, 1 through 20. Each multiple choice question is worth 3.5 points giving a total of 70 points. The second part is the two hand graded questions, problems 21 & 22, each is worth 15 points. Please check to see that your exam has all 22 questions. For the first part you will need a Pencil to make marks on your answer card, if you do not have a pencil, please ask to borrow one from your proctor.

Write your ID NUMBER (not your SS #) at the top of your answer card, using one blank for each of the six digits. Shade in the corresponding boxes below. Also print your name at the top of your card and print your name on problems 21, 22.

Since it is hard to erase a pencil mark off of the card make sure you have come to your final conclusion before you mark the answer on the card. Also note that problem 22 appears on both sides of the paper, don't forget to do 22, part c, on the other side.

1) Find the distance (shortest distance) from the point \((2, -3, 5)\) to the plane \(x = 1\).

(A) 1  B) \(\sqrt{2}\)  C) 2  D) \(\sqrt{3}\)  E) 4  F) \(\sqrt{5}\)  G) 5  H) 6  I) \(\frac{3}{2}\)  J) \(\frac{2}{3}\)

closest point to \((a, -3, 5)\) on the plane is \((1, -3, 5)\). Distance is then 1.
2) The vector equation for the line passing through the point \((2, 4, 5)\) which is perpendicular to the plane \(3x + 7y - 5z = 21\) is given by \(\mathbf{r}(t) = \) 

\[
\begin{align*}
A) & \quad <3 + 2t, 7 + 4t, -5 + 5t> \\
B) & \quad <2 + 7t, 3 + 5t, -5 + 7t> \\
C) & \quad <4 + 5t, -5 + 7t, 4 + 3t> \\
D) & \quad <5 + 4t, 2 - 5t, 7 + 3t> \\
E) & \quad <3 + 7t, -5 + 2t, 4 + 5t> \\
F) & \quad <2 + 3t, 4 + 7t, 5 - 5t> \\
G) & \quad <7 + 3t, 2 - 5t, 5 + 4t> \\
H) & \quad <2 - 5t, 7 + 3t, 5 + 4t> \\
I) & \quad <3t, 7t, -5t> \\
J) & \quad <2t, 4t, 5t>
\end{align*}
\]

Hence the line is given by \(x = 2 + 3t, \ y = 4 + 7t, \ z = 5 - 5t\).

3) Find the vector projection of \(\mathbf{b} = 4\mathbf{i} - \mathbf{j} + 4\mathbf{k}\) onto \(\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}\).

\[
\begin{align*}
\text{proj}_\mathbf{a} \mathbf{b} & = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||^2} \mathbf{a} \\
\mathbf{a} \cdot \mathbf{b} & = <1, -1, 1> \cdot <4, -1, 4> \\
& = 9 \\
||\mathbf{a}|| & = \sqrt{3} \\
\text{so} \quad ||\mathbf{a}|| & = 3
\end{align*}
\]

\[
\begin{align*}
giving \quad \frac{9}{3} & = <1, -1, 1> \\
& = <3, -3, 3>
\end{align*}
\]

\[
\begin{align*}
\text{proj}_\mathbf{a} \mathbf{b} & = \frac{\mathbf{a} \cdot \mathbf{b}}{||\mathbf{a}||^2} \mathbf{a} \\
& = \frac{9}{3} \mathbf{a} \\
& = <3, -3, 3>
\end{align*}
\]
4) If \( \mathbf{a} = \mathbf{i} + \mathbf{j} \) and \( \mathbf{b} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k} \), then in radians, the angle between them is:

\[
\text{If } \theta \text{ is angle, then we have}
\]

\[
\cos(\theta) = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} = \frac{3}{\sqrt{2} \sqrt{14}} = \frac{3}{\sqrt{28}}
\]

\[\theta = \arccos\left(\frac{3}{\sqrt{28}}\right) \approx 0.968\]

A) 0.541  
B) 0.634  
C) 0.659  
D) 0.701  
E) 0.782  
F) 0.841  
G) 0.904  
H) 0.968  
I) 1.101  
J) 1.152  
K) 2.9281

5) Find symmetric equations for the line through the point \((0, 2, -1)\) which is parallel to the line \(x = 1 + 2t, y = 3t, z = 5 - 7t\).

(A) \( \frac{x-2}{3} = \frac{y-3}{3} = \frac{z+1}{3} \)  
B) \( \frac{x-1}{2} = \frac{y-2}{3} = \frac{z+7}{3} \)  
C) \( \frac{x-1}{3} = \frac{y-2}{5} = \frac{z-7}{3} \)  
D) \( \frac{x+2}{3} = \frac{y+3}{2} = \frac{z+1}{2} \)  
E) \( \frac{x+1}{3} = \frac{y-1}{2} = \frac{z+3}{5} \)  
F) \( \frac{x+2}{2} = \frac{y-3}{2} + \frac{z-5}{3} \)  
G) \( \frac{x+2}{3} = \frac{y+1}{4} = \frac{z-2}{1} \)  
H) \( \frac{x-7}{3} = \frac{y+3}{4} = \frac{z-2}{1} \)  
I) \( \frac{x+1}{2} = \frac{y+2}{5} = \frac{z-1}{5} \)  
J) \( \frac{x}{1} = \frac{y-2}{3} = \frac{z-1}{5} \)

Since line is parallel, we have direction vector is <2, 3, -7> so:

\[x = 2t, \quad y = 2+3t, \quad z = -1-7t\]

\[
\frac{x-0}{2} = \frac{y-2}{3} = \frac{z+1}{-7}
\]
6) Find the number \( b \) given that the line through \((0, 1, 1)\) and \((1, -1, 6)\) has its direction vector orthogonal to the direction vector of the line through \((-4, b, 1)\) and \((-1, 6, 2)\) (i.e. the lines are perpendicular).

\[
\begin{align*}
A) \ 1 & \quad B) \ -1 & \quad C) \ 2 & \quad D) \ -2 & \quad E) \ 3 & \quad F) \ -3 & \quad G) \ 4 & \quad H) \ -4 & \quad I) \ 5 & \quad J) \ -5 \\
\text{line through } (0, 1, 1), (1, -1, 6) & \text{ has direction vector } <1, -2, 5> \\
\text{line through } (-4, b, 1), (-1, 6, 2) & \text{ has direction vector } <3, 6-b, 1> \\
\text{perpendicular means } & \quad 0 = <1, -2, 5> \cdot <3, 6-b, 1> \\
& \quad = 2b - 4 \\
& \quad \Rightarrow b = 2
\end{align*}
\]

7) Find the area of the parallelogram with vertices at the points

\[A(-1, 1), \quad B(1, 4), \quad C(5, 2), \quad \text{and} \quad D(3, -1).\]

\[
\begin{align*}
A) \ 6 & \quad B) \ 8 & \quad C) \ 10 & \quad D) \ 12 & \quad E) \ 14 & \quad F) \ 16 & \quad G) \ 18 & \quad H) \ 20 & \quad I) \ 22 & \quad J) \ 22
\end{align*}
\]

\[
\begin{align*}
\vec{AB} & = <2, 3> \\
\vec{AD} & = <4, -2> \\
\text{Area} & = \begin{vmatrix}
\vec{a} & \vec{j} & \vec{k} \\
2 & 3 & 0 \\
4 & -2 & 0
\end{vmatrix} = \begin{vmatrix}
-16 & \vec{k}
\end{vmatrix} = 16
\end{align*}
\]
8) Find an equation for the plane that contains the line \( \mathbf{r}(t) = t \mathbf{i} + 2t \mathbf{j} + 3t \mathbf{k} \) and also contains the point \((1, -1, 1) = \mathbf{c}\):

A) \(2x + 5y - z + 4 = 0\)
B) \(x + 2y - 3z + 4 = 0\)
C) \(x - 2y - 3z = 0\)
D) \(-2x + y + 4z - 1 = 0\)
E) \(5x + 2y - 3z = 0\)
F) \(2x + 3y - 6z - 4 = 0\)
G) \(x + y + z - 1 = 0\)
H) \(-x + y - z + 3 = 0\)
I) \(2x - 4y + 3z + 2 = 0\)
J) \(3x - y + 4z = 0\)

Point on line
\[ t = 0 : \quad (0, 0, 0) = \mathbf{A} \]
\[ t = 1 : \quad (1, 2, 3) = \mathbf{B} \]

Normal line to plane, would be
\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
1 & 2 & 3 \\
1 & -1 & 1 \\
\end{vmatrix}
= 5\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}
\]

Using \( (0, 0, 0) \) we get
\[ 5x + 2y - 3z = 0 \]
9) Find an equation for the plane made up of all points which are **equidistant** from the points \((1, 1, 1)\) and \((-1, -1, -1)\).

A) \(2x + y + z = 0\)
B) \(x + 2y + 2z = 0\)
C) \(x + 2y + z = 0\)
D) \(x + y + z = 0\)
E) \(2x + 2y + z = 0\)
F) \(x + 2y + 2z - 4 = 0\)
G) \(x - y + z + 2 = 0\)
H) \(x - 2y + z - 8 = 0\)
I) \(2x - y + z + 3 = 0\)
J) \(x - 2y + z + 1 = 0\)

If \((x, y, z)\) is on plane

\[\text{then} \quad (x-1)^2 + (y-1)^2 + (z-1)^2 = (x+1)^2 + (y+1)^2 + (z+1)^2\]

\[x^2 - 2x + 1 + y^2 - 2y + 1 + z^2 - 2z + 1 = x^2 + 2x + 1 + y^2 + 2y + 1 + z^2 + 2z + 1\]

\[y \in \text{dim}\]

\[-2x - 2y - 2z = 2x + 2y + 2z\]

\[4(x + y + z) = 0\]

\[\text{or} \quad x + y + z = 0\]
10) Find the distance between the two parallel planes $x + 2y + 2z = 5$ and $3x + 6y + 6z = 4$

A) $\frac{7}{3}$  B) $\frac{8}{3}$  C) $\frac{13}{3}$  D) $\frac{11}{5}$  E) $\frac{2}{5}$  F) $\frac{3}{5}$  G) $\frac{11}{10}$  H) $\frac{5}{9}$  I) $\frac{5}{9}$  J) $\frac{6}{9}$

Point on $x + 2y + 2z = 5$ is $(1, 1, 1) = P$

Distance from $(1, 1, 1)$ to $3x + 6y + 6z = 4$ is gotten by finding point $Q$ on plane. For $y = z = 0$ we get $Q \left( \frac{4}{3}, 0, 0 \right)$.

Formula for distance (by projection)

$$d = \frac{|\overrightarrow{PQ} \cdot \overrightarrow{n}|}{|\overrightarrow{n}|}$$

where $\overrightarrow{n}$ is normal vector $\overrightarrow{n} = \langle 3, 6, 6 \rangle$

$$|\overrightarrow{n}| = \sqrt{81} = 9$$

$$\overrightarrow{PQ} \cdot \overrightarrow{n} = -11$$

$$\frac{|\overrightarrow{PQ} \cdot \overrightarrow{n}|}{|\overrightarrow{n}|} = \frac{11}{9}$$
11) Which of the following surfaces would have hyperbolas and parabolas for their horizontal and vertical traces.

A) \( z^2 = x^2 + y^2 \)
B) \( z^2 = x^2 - y^2 \)
C) \( z = x^2 + y^2 \)
D) \( z = x^2 - y^2 \)
E) \( z = x - y^2 \)
F) \( z = x + y^2 \)
G) \( z^2 = 2x^2 - y^2 \)
H) \( z^2 = x - y^2 \)
I) \( z^2 = -x^2 + y \)
J) \( z^2 = 2x + y \)

\[ z = k \]
\[ x^2 - y^2 = k \]
\[ x = k \]
\[ x^2 - y^2 = k \]
\[ y = k \]
\[ x^2 - y^2 = k \]

are both parabolas

12) A list of all the horizontal and vertical traces for the surface

\( f(x, y) = x - y^2 \) is:

A) ellipses
B) parabolas
C) hyperbolas
D) lines
E) ellipses and parabolas
F) parabolas and hyperbolas
G) hyperbolas and lines
H) parabolas and lines
I) ellipses and hyperbolas
J) ellipses and lines

\[ z = f(x, y) \]
\[ z = k \]
\[ x = y^2 + k \]
\[ x = k \]
\[ z = -y^2 + k \]
both are parabolas
\[ y = k \]
\[ z = x - k^2 \]

line
13) Which of the following vector functions (space curves) lies on the intersection of the cylinder \( x^2 + y^2 = 1 \) and the plane \( y + z = 2 \)?

A) \( \langle \cos(t), \sin(t), \cos^2(t) \rangle \)

B) \( \langle \sin(t), \cos(t), \tan(t) \rangle \)

C) \( \langle \cos(t), \sin(t), 2 - \sin(t) \rangle \)

D) \( \langle \sin(t), \cos(t), 2 - \sin(t) \rangle \)

E) \( \langle \cos(t), \sin(t), \sin(2t) \rangle \)

F) \( \langle \sin(t), \cos(t), \sin(2t) \rangle \)

G) \( \langle \cos(t), \sin(t), 2 \sin(t) \rangle \)

H) \( \langle \sin^2(t), \cos^2(t), 2 - \cos^2(t) \rangle \)

I) \( \langle \sin(t), \sin^2(t), 2 + \sin^2(t) \rangle \)

J) \( \langle 2 \sin(t), \cos(t), \sin^2(t) \rangle \)

\[ x = \cos(t) \]
\[ y = \sin(t) \]
\[ z = 2 - \sin(t) \]
\[ x^2 + y^2 = 1 \]
\[ y + z = 2 \]

14) If \( \mathbf{r}(t) = \langle \sin(t), \cos(t), t \rangle \) then find the tangent vector \( \mathbf{T}(\frac{\pi}{4}) \).

A) \( \langle \sqrt{2}, -\sqrt{2}, 1 \rangle \)

B) \( \langle \sqrt{2}, -\sqrt{2}, \frac{1}{2} \rangle \)

C) \( \langle -\frac{1}{2}, -\frac{1}{2}, \frac{\sqrt{2}}{2} \rangle \)

D) \( \langle \frac{1}{2}, -\frac{1}{2}, \frac{\sqrt{2}}{2} \rangle \)

E) \( \langle \frac{1}{2}, -\frac{1}{2}, 1 \rangle \)

F) \( \langle -\sqrt{2}, \sqrt{2}, \frac{1}{2} \rangle \)

G) \( \langle -\frac{1}{2}, \frac{1}{2}, 1 \rangle \)

H) \( \langle -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{2} \rangle \)

I) \( \langle \sqrt{2}, -\sqrt{2}, \frac{1}{2} \rangle \)

J) \( \langle 2, -2, \frac{1}{2} \rangle \)

\[ \mathbf{r}'(t) = \langle \cos(\pi/4), -\sin(\pi/4), 1 \rangle \]

\[ |\mathbf{r}'(t)| = \sqrt{\cos^2(\pi/4) + \sin^2(\pi/4)} + 1 = \sqrt{2} \]

\[ \mathbf{T}(t) = \langle \frac{\cos(t)}{\sqrt{2}}, -\frac{\sin(t)}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle \]

\[ \mathbf{T}(\frac{\pi}{4}) = \langle \frac{1}{2}, -\frac{1}{2}, \sqrt{2} \rangle \]

\[ \sin(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) = \frac{1}{\sqrt{2}} \]
15) Find symmetric equations for the tangent line to \( \mathbf{r}(t) = \langle t^2 - 1, t^2 + t, t + 1 \rangle \) at the point \((0, 2, 2)\).

\[
\mathbf{r}'(t) = \langle 2t - 1, 2t + 1, 1 \rangle
\]

At \( t = 1 \),

\[
\mathbf{p}^t (0, 2, 2) \quad \mathbf{t}
\]

\[
\mathbf{r}'(1) = \langle 1, 3, 1 \rangle
\]

So, \( \mathbf{p}^0 \)

\[
X = \tau, \quad Y = 2 + 3\tau, \quad Z = 2 + \tau
\]

\[
\frac{X - 1}{1} = \frac{Y - 2}{3} = \frac{Z - 2}{1}
\]

16) Find the arc length of the curve \( \mathbf{r}(t) = \langle \sin(t), \cos(t), 2t \rangle \) from the point \((0, 0, 0)\) to the point \((0, -1, 2\pi)\).

\[
\mathbf{r}'(t) = \langle \cos(t), -\sin(t), 2 \rangle
\]

\[
|\mathbf{r}'(t)| = \sqrt{\cos^2(t) + \sin^2(t) + 4} = \sqrt{5}
\]

\((0, 1, 0)\) is \( t = 0 \)

\((0, -1, 2\pi)\) \( t = \pi \)

\[
l = \int_0^\pi \sqrt{5} \, dt = \sqrt{5} \left|_0^\pi \right. = \sqrt{5} \pi
\]
17) If a particle has a position function given by \( \mathbf{r}(t) = i + 2t j + t^2 k \), then find \( a_r \), the **tangential component** of acceleration (recall: \( \mathbf{a} = a_r \mathbf{T} + a_n \mathbf{N} \)).

\[
\begin{align*}
\text{Fact: } q_{\frac{r}{T}} &= v'(t) \\
\text{where } v &= |\mathbf{r}'(t)| \\
\mathbf{r}'(t) &= \begin{pmatrix} 0 \\ 2 \end{pmatrix} \\
|\mathbf{r}'(t)| &= \sqrt{4 + 4t^2} \\
&= 2 \sqrt{1 + t^2} \\
&= 2 \sqrt{1 + t^2} \quad \text{(option E)}
\end{align*}
\]

\[
q_{\frac{r}{T}} = \frac{v'}{v} = \frac{2t}{2 \sqrt{1 + t^2}} = \frac{t}{\sqrt{1 + t^2}}
\]
18) Find the curvature of \( \mathbf{r}(t) = t \mathbf{i} + t^2 \mathbf{j} + t \mathbf{k} \) at \( t = 2 \).

\[
\kappa(t) = \frac{1}{\left|\mathbf{r}'(t) \times \mathbf{r}''(t)\right|} \frac{1}{\left|\mathbf{r}'(t)\right|^3}
\]

\[
\mathbf{r}'(t) = \langle 1, 2t, 1 \rangle
\]
\[
\mathbf{r}''(t) = \langle 0, 2, 0 \rangle
\]

At \( t = 2 \), we set

\[
\frac{\mathbf{r}'(t) \times \mathbf{r}''(t)}{\left|\mathbf{r}'(t) \times \mathbf{r}''(t)\right|} = \frac{1}{2} \mathbf{i} + \frac{1}{2} \mathbf{j} + \frac{1}{2} \mathbf{k}
\]

\[
\left|\mathbf{r}'(t) \times \mathbf{r}''(t)\right| = \sqrt{8}
\]

\[
\mathbf{r}'(2) = \langle 1, 4, 1 \rangle
\]
\[
\left|\mathbf{r}'(2)\right| = \sqrt{18}
\]

\[
\frac{\sqrt{8}}{\left(18\right)^{3/2}} = \frac{2 \sqrt{2}}{9 \sqrt{2} \cdot 2^{3/2}} = \frac{2 \sqrt{2}}{27 \cdot 2^{3/2}} = \frac{1}{27}
\]

Hence, \( \kappa(t) = \frac{1}{27} \).
19) If \( r(t) = 2 \sin(t) \mathbf{i} + 5t \mathbf{j} + 2 \cos(t) \mathbf{k} \), find the binormal vector at \( t = \frac{\pi}{2} \).

\( \text{i.e. } \mathbf{B} \left( \frac{\pi}{2} \right) \)

\[
\begin{align*}
A) & \quad \frac{3}{\sqrt{29}} \mathbf{i} - \frac{2}{\sqrt{29}} \mathbf{j} + \frac{5}{\sqrt{29}} \mathbf{k} \\
B) & \quad \frac{3}{\sqrt{29}} \mathbf{i} + \frac{5}{\sqrt{29}} \mathbf{k} \\
C) & \quad \frac{6}{\sqrt{29}} \mathbf{i} - \frac{5}{\sqrt{29}} \mathbf{j} \\
D) & \quad \frac{2}{\sqrt{29}} \mathbf{j} + \frac{5}{\sqrt{29}} \mathbf{k} \\
E) & \quad \frac{2}{\sqrt{21}} \mathbf{i} - \frac{3}{\sqrt{21}} \mathbf{j} + \frac{4}{\sqrt{21}} \mathbf{k} \\
F) & \quad \frac{3}{\sqrt{29}} \mathbf{i} + \frac{2}{\sqrt{29}} \mathbf{j} + \frac{5}{\sqrt{29}} \mathbf{k} \\
G) & \quad \frac{2}{\sqrt{21}} \mathbf{j} + \frac{5}{\sqrt{21}} \mathbf{k} \\
H) & \quad \frac{3}{\sqrt{29}} \mathbf{i} - \frac{5}{\sqrt{29}} \mathbf{j} + \frac{7}{\sqrt{29}} \mathbf{k} \\
I) & \quad \frac{7}{\sqrt{29}} \mathbf{i} - \frac{9}{\sqrt{29}} \mathbf{j} + \frac{5}{\sqrt{29}} \mathbf{k} \\
J) & \quad \frac{7}{\sqrt{29}} \mathbf{i} - \frac{9}{\sqrt{29}} \mathbf{j} + \frac{5}{\sqrt{29}} \mathbf{k}
\end{align*}
\]

\[
\begin{align*}
\sqrt{\mathbf{r}'(t)} &= \sqrt{2 \cos(t), 5, -2 \sin(t)} \\
\|\mathbf{r}'(t)\| &= \sqrt{4 + 25} = \sqrt{29}
\end{align*}
\]

\[
\mathbf{r}'(t) = \left< 2 \cos(t), 5, -2 \sin(t) \right>
\]

\[
\mathbf{T}(\frac{\pi}{2}) = \left< \frac{2}{\sqrt{29}}, \frac{5}{\sqrt{29}}, -\frac{2}{\sqrt{29}} \right>
\]

\[
\mathbf{T}'(\frac{\pi}{2}) = \left< -\frac{2}{\sqrt{29}}, 0, -\frac{2}{\sqrt{29}} \right>
\]

\[
\mathbf{N}(\frac{\pi}{2}) = \mathbf{T}(\frac{\pi}{2}) \times \mathbf{T}'(\frac{\pi}{2}) = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & \frac{5}{\sqrt{29}} & -\frac{2}{\sqrt{29}} \\
-1 & 0 & 0
\end{vmatrix}
\]

\[
= \left< \frac{2}{\sqrt{29}} \mathbf{j} + \frac{5}{\sqrt{29}} \mathbf{k} \right>
\]

\[
\mathbf{\beta}(\frac{\pi}{2}) = \mathbf{T}(\frac{\pi}{2}) \times \mathbf{N}(\frac{\pi}{2})
\]
20) The position of a particle is given by \( \mathbf{r}(t) = t^2 \mathbf{i} + 5t \mathbf{j} + (t^2 - 16t) \mathbf{k} \).
At what time (value of \( t \)) will its speed be a minimum?

A) 2  B) \( \sqrt{3} \)  C) 4  D) \( \sqrt{5} \)  E) 6  F) 7  G) \( \sqrt{8} \)  H) 9  I) 10  J) \( \sqrt{11} \)

\[
\mathbf{v}'(t) = \langle 2t, 5, 2t - 16 \rangle
\]
\[
|\mathbf{v}'(t)| = \sqrt{8t^2 - 64t + 256}
\]

To find minimum we take derivative and set it to zero.

\[
|\mathbf{v}'(t)| = \frac{16t - 64}{2 \sqrt{8t^2 - 64t + 256}} = 0
\]

Only happen when \( t = 4 \)

\[
\mathbf{v}' = \frac{-1}{4} + \frac{1}{16}
\]

Here \( t = 4 \) is a br minimum
21) Given the two planes \( L_1 : x + y + 2z - 4 = 0 \) and \( L_2 : x + 2y - z - 2 = 0 \).

a) Find the angle (smaller angle) between the two planes. (5 points)

The normal vectors are given by

\[ \vec{n}_1 = i + j + 2k \]
[\( \vec{n}_2 = i + 2j - k \)]

The cosine of the angle between the normal vectors is

\[ \cos(\Theta) = \frac{\vec{n}_1 \cdot \vec{n}_2}{||\vec{n}_1|| ||\vec{n}_2||} = \frac{1}{\sqrt{6} \sqrt{6}} = \frac{1}{6} \]

\[ \Theta = \arccos \left( \frac{1}{6} \right) \approx 1.40 \text{ radians} \]

b) Find some vector which lies in both planes. (5 points)

Vectors in both planes are vectors orthogonal to both normal vectors. To be in both planes we need a vector orthogonal to \( \vec{n}_1 \) and \( \vec{n}_2 \). This is

\[ \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix}
    i & j & k \\
    1 & 1 & 2 \\
    1 & 2 & -1 \\
\end{vmatrix} = -5i + 3j + k \]

This vector is orthogonal to both normals.

c) Find a formula for the line of intersection of the two planes. (5 pts)

From part (b) we know that

\[-5i + 3j + k \text{ is the direction vector of the line} \]

We need a point on the line. From part (b) we know that

\[ x + y + 2z = 4 \] (let \( z = 0 \) then we get \( x = 6 \))
\[ x + 2y - z = 2 \] (get \( x = 6 \))

The point is \( P = (6, -2, 0) \)

Parameter equation:

\[ x = 6 - 5t \\
\[ y = -2 + 3t \\
\[ z = t \]
NAME: [redacted]  ID #: [redacted]

Show the work you did to find answers. Write your name and ID# clearly.

22) If \( \mathbf{r}(t) = \sin(t) \mathbf{i} - \cos(t) \mathbf{j} + 2t \mathbf{k} \).

a) Find the arc length of the curve from the point \((0, -1, 0)\) to the point \((1, 0, \pi)\). (5 pts)

\[
\mathbf{r}'(t) = \langle \cos(t), \sin(t), 2 \rangle
\]

\[
|\mathbf{r}'(t)| = \sqrt{5}
\]

\[
L = \int_{0}^{\pi} \sqrt{5} \, dt = \frac{\sqrt{5} \pi}{2}
\]

b) Reparametrize the curve with respect to the arc length measurement starting at \( t = 0 \) in the direction of increasing \( t \). (i.e. write formulas for \( \mathbf{r}(s) \)). (5 points)

\[
S = \int_{0}^{\tau} \sqrt{5} \, du = \sqrt{5} \tau \quad \text{so} \quad \tau = \frac{S}{\sqrt{5}}
\]

and

\[
\mathbf{r}(s) = S \sin \left( \frac{s}{\sqrt{5}} \right) \mathbf{i} - \cos \left( \frac{s}{\sqrt{5}} \right) \mathbf{j} + \frac{2s}{\sqrt{5}} \mathbf{k}
\]

The rest of the problem, part c, is on the other side. Turn over.
c) Find the osculating plane at the point $(1, 0, \pi)$. (5 points)

The point is when $t = \frac{\pi}{2}$.

\[
\mathbf{r}'(t) = \left< \cos(t), \sin(t), 2 \right>
\]

\[
\mathbf{r}'\left(\frac{\pi}{2}\right) = \left< 0, 1, 2 \right>, \quad |\mathbf{r}'(t)| = \sqrt{5}
\]

So, \[
\mathbf{T}(t) = \left< \frac{\cos(t)}{\sqrt{5}}, \frac{\sin(t)}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right>
\]

and \[
\mathbf{T}\left(\frac{\pi}{2}\right) = \left< 0, \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right>
\]

\[
\mathbf{T}'(t) = \left< \frac{\sin(t)}{\sqrt{5}}, \frac{\cos(t)}{\sqrt{5}}, 0 \right>
\]

\[
\mathbf{T}'\left(\frac{\pi}{2}\right) = \left< -\frac{1}{\sqrt{5}}, 0, 0 \right>
\]

So, \[
\mathbf{N}(\frac{\pi}{2}) = \left< -1, 0, 0 \right>
\]

The normal vector to the plane would be \[
\mathbf{T}\left(\frac{\pi}{2}\right) \times \mathbf{N}(\frac{\pi}{2})
\]

\[
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\
-1 & 0 & 0 \\
\end{vmatrix} = -\frac{2}{\sqrt{5}} \mathbf{j} + \frac{1}{\sqrt{5}} \mathbf{k}
\]

The osculating plane is:

\[-\frac{2}{\sqrt{5}} (y - 0) + \frac{1}{\sqrt{5}} (z - \pi) = 0 \quad \text{or}\]

\[-\frac{2}{\sqrt{5}} y + \frac{1}{\sqrt{5}} z - \frac{\pi}{\sqrt{5}} = 0\]