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J.E. Pascoe

July 24, 2014

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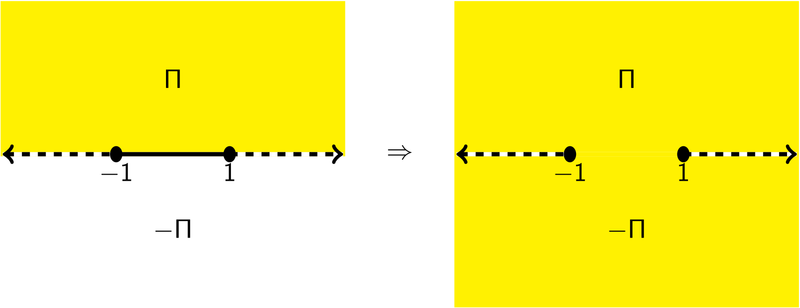

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We would like to say that "f analytically continues to $\Pi^2 \cup (-1,1)^2 \cup -\Pi^2$" by analogy with the reflection principle. However, $\Pi^2 \cup (-1,1)^2 \cup -\Pi^2$ is *not an open set;* so it is unclear what saying "analytically continues" would mean.

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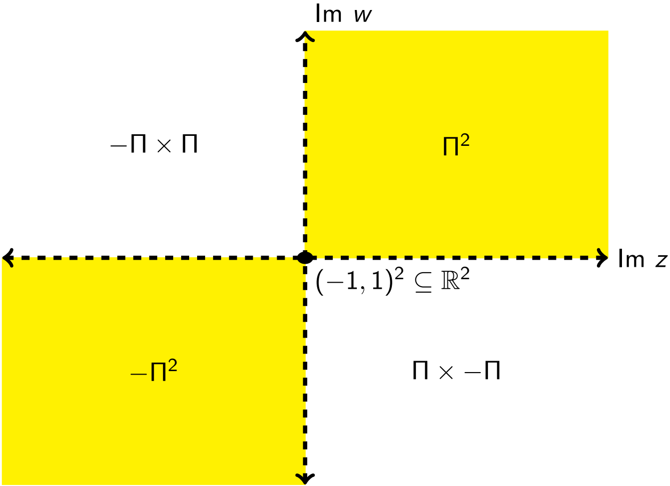

Figure : A diagram of $\Pi^2 \cup (-1,1)^2 \cup -\Pi^2$ projected onto the imaginary axes. Note that this set is not open.

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- The edge-of-the-wedge theorem was discovered by physicist Nikolay Bogoliubov.
- He proved the theorem to show that "Wightman functions" which arise in some formulation of quantum field theory have nice analytic continuation properties.
- The theorem's importance as a stem theorem in several complex variables was realized over time. Rudin wrote an excellent text on the subject, called *Lectures on the edge-of-the-wedge theorem.*

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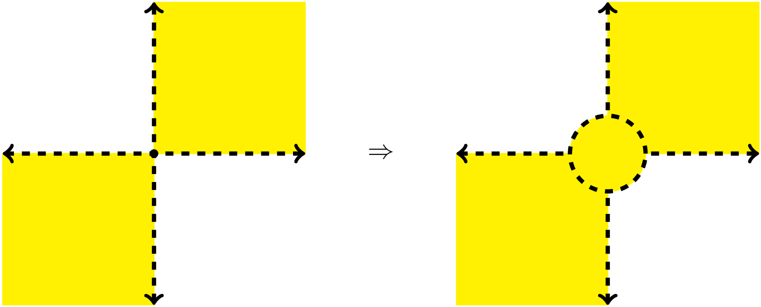

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- Charles Löwner showed that a function $f : (-1, 1) \to \mathbb{R}$ is *matrix monotone* in the sense that for any two self-adjoint matrices A, B with spectrum in $(-1, 1)$, such that

$$A \leq B \Rightarrow f(A) \leq f(B)$$

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- Agler, McCarthy and Young showed that an analogue of Löwner's holds for Pick functions in several variables.
- P. showed that a phenomenon similar to the edge-of-the-wedge theorem holds for the boundary values of Pick functions- a *wedge-of-the-edge theorem.*

The wedge-of-the-edge theorem (P.)

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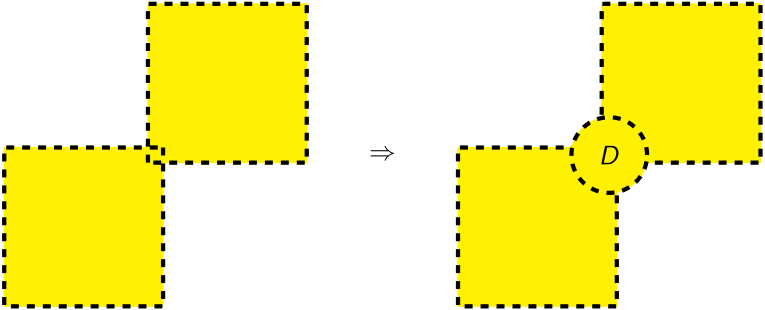
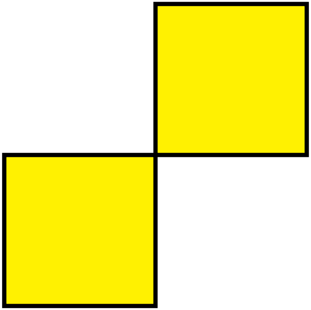


Example: The geometric mean $\sqrt{xy}$



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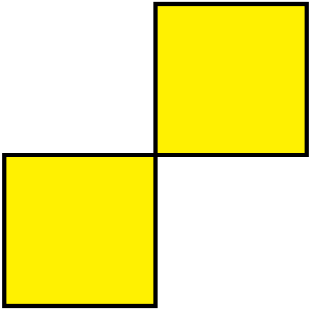

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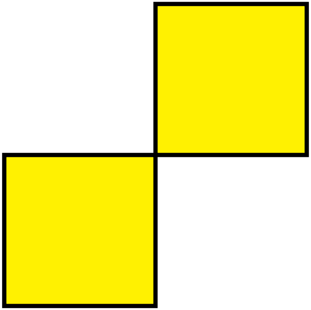

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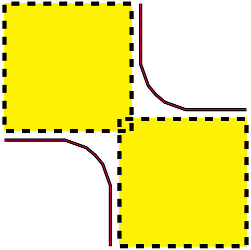
Example: A rational function with singularities on the hyperbola $txy = 1$



The function for any t, the function $\frac{x}{1-txy}$ defines a Pick function. Furthermore, each of these is real-valued on $(-\varepsilon, 1) \times (-1, \varepsilon) \cup (-1, \varepsilon) \times (-\varepsilon, 1)$ for small enough ε. However, for large t, the singular set of these function approach to being the x and y axes.

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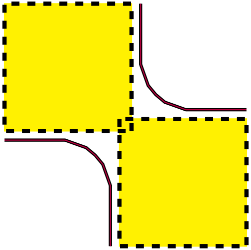

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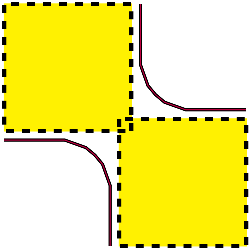

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A detailed wedge-of-the-edge theorem (P.)

Theorem

For any Pick function in two variables f which has a continuous real-valued extension to $(-1,\varepsilon)^2 \cup (-\varepsilon,1)^2$,

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The above is a special case of the Julia inequality obtained by Agler, McCarthy, and Young on the bidisk transformed to be an inequality on Π^2. On the other hand, it can be obtained using realization theory, specifically the Nevanlinna representations obtained by Agler, Tully-Doyle and Young.

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An application: relaxing the Agler-McCarthy-Young Löwner theorem in several variables

Agler, McCarthy and Young showed that any differentiable matrix monotone function on $(0,1)^2$ analytically continues to Π^2 as a Pick function. Recently, we showed that the differentiability hypothesis can be dropped using inequalities arising from the wedge-of-the-edge theorem by:

- mollifying such a function (A relaxation technique which "blurs" a function, making it differentiable and subject to the aforementioned theorem.)
- As we mollify the function less and less, we recover the original function. So, applying normal families argument, we can show that the original function was differentiable.

That is, any matrix monotone function on $(0,1)^2$ analytically continues to Π^2 as a Pick function.

A wedge of the edge of the wedge

J.E. Pascoe

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