Math 233 - Exam I - Spring 2013

February 6, 2013 - Rachel Roberts

NAME:

SOLUTIONS

STUDENT ID NUMBER:

General instructions: This exam has 16 questions, each worth the same amount. Check that no pages are missing and notify your proctor if you detect any problems with your copy of the exam. Mark your ID number on the six blank lines on the top of your answer card, using one line for each digit. Print your name on the top of the card. Choose the answer that is closest to the solution and mark your answer card with a PENCIL by shading in the correct box. You may use a 3× 5 card with notes and any calculator that does not have graphing functions. GOOD LUCK!

1. Suppose you are given the following points:

   $A(100, -\underline{1}, 22), B(205, \underline{1/2}, 3), C(55, -\underline{100}, 24), D(5, \underline{11}, 99), E(1001, \underline{299}, -56)$.

   List these points in order of increasing distance from the xz-plane.

   (A) A, B, C, D, E
   (B) B, A, C, D, E
   (C) B, A, D, C, E
   (D) A, B, D, C, E
   (E) B, A, E, D, C
   (F) D, C, A, B, E
   (G) B, A, C, E, D
   (H) C, A, B, D, E

   ie: list in increasing $|y|$ - value

   B, A, D, C, E

2. Find $|\mathbf{v} - \mathbf{w}|$, when $\mathbf{v} = \langle 2, 3, 5 \rangle$ and $\mathbf{w} = \langle -3, 5, 2 \rangle$.

(A) 7
(B) -8
(C) 17
(D) 38
(E) $\sqrt{5}$
(F) $\sqrt{7}$
(G) $\sqrt{17}$
(H) $\sqrt{38}$

$v - w = \langle +5, -2, +3 \rangle$

$|v - w| = \sqrt{25 + 4 + 9} = \sqrt{38}$

H

3. Which of the following expressions are meaningful?

(a) $\mathbf{a} \cdot (\mathbf{b} \cdot \mathbf{c})$

(b) $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

(c) $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$

(d) $\mathbf{a} \cdot \mathbf{b} \cdot \mathbf{c}$

(e) $\mathbf{a} \cdot |\mathbf{b} + \mathbf{c}|$

(f) $(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$

(g) $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$

(h) $\mathbf{a} \times (\mathbf{b} \times \mathbf{c})$

(i) $(\mathbf{a} \cdot \mathbf{b}) \times (\mathbf{c} \cdot \mathbf{d})$

(j) $(\mathbf{a} \times \mathbf{b}) \cdot (\mathbf{c} \times \mathbf{d})$

(A) a,b,d,f,j

(B) b,c,f,h,j

(C) c,f,h,j

(D) b,c,h,j

(E) b,f,h,i,j

(F) c,f,g,h,j

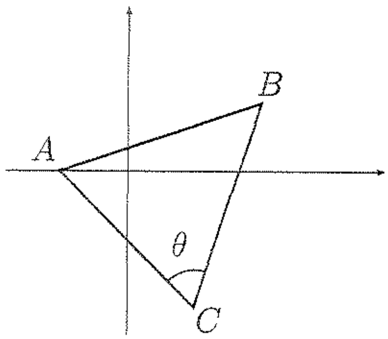
(G) c,e,f,h

(H) a,c,h,i,j

B

4. Find the angle θ in the triangle of the figure whose vertices are

$$A = (-1, 0), \quad B = (2, 1), \quad C = (1, -2).$$



(A) $cos^{-1}(\frac{1}{\sqrt{5}})$

(B) $cos^{-1}(\frac{1}{\sqrt{80}})$

(C) $cos^{-1}(\frac{4}{\sqrt{10}})$

(D) $cos^{-1}(\frac{2}{\sqrt{10}})$

(E) $cos^{-1}(\frac{4}{1})$

(F) $cos^{-1}(\frac{1}{4})$

(G) $cos^{-1}(\frac{\sqrt{2}}{2})$

(H) $cos^{-1}(\frac{\sqrt{3}}{2})$

$\vec{CA} = \langle -2, 2 \rangle \qquad |\vec{CA}| = \sqrt{8} = 2\sqrt{2}$

$\vec{CB} = \langle 1, 3 \rangle \qquad |\vec{CB}| = \sqrt{1+9} = \sqrt{10}$

$\vec{CA} \cdot \vec{CB} = -2 + 6 = 4$

$\vec{CA} \cdot \vec{CB} = |\vec{CA}||\vec{CB}| \cos\theta$

$4 = 2\sqrt{20} \quad \cos\theta = 4\sqrt{5} \cos\theta$

$\therefore \cos\theta = \frac{1}{\sqrt{5}}$

$\theta = \cos^{-1}(\frac{1}{\sqrt{5}})$

A

5. Find the vector projection $\text{proj}_{\mathbf{v}}\mathbf{u}$, where $\mathbf{v} = 5\mathbf{i} + \mathbf{j}$ and $\mathbf{u} = 2\mathbf{i} + 3\mathbf{j}$.

(A) $\frac{1}{2}(5\mathbf{i} + \mathbf{j})$

(B) $\frac{1}{\sqrt{2}}(5\mathbf{i} + \mathbf{j})$

(C) $\frac{1}{\sqrt{26}}(5\mathbf{i} + \mathbf{j})$

(D) $\sqrt{6}(5\mathbf{i} + \mathbf{j})$

(E) $2\mathbf{i} + 3\mathbf{j}$

(F) $\frac{1}{\sqrt{13}}(2\mathbf{i} + 3\mathbf{j})$

(G) $13(2\mathbf{i} + 3\mathbf{j})$

(H) $\sqrt{13}(2\mathbf{i} + 3\mathbf{j})$

$$\text{proj}_v u = \frac{u \cdot v}{|v|^2} \cdot v \qquad |v| = \sqrt{26}$$

$$= \frac{10 + 3}{26} (5i + j)$$

$$= \frac{1}{2} (5i + j)$$

A

6. Which vector is perpendicular to both $\langle 1, 2, 3\rangle$ and $\langle -2, 5, 1\rangle$.

(A) $\langle -13, 7, 9\rangle$
(B) $\langle -26, 7, 9\rangle$
(C) $\langle -26, -14, 18\rangle$
(D) $\langle 104, 27, -3\rangle$
(E) $\langle 25, 102, -1\rangle$
(F) $\langle 43, -17, -22\rangle$
(G) $\langle -43, -17, 22\rangle$
(H) $\langle -41, -5, 73\rangle$

$$\begin{vmatrix} i & j & k \\ 1 & 2 & 3 \\ -2 & 5 & 1 \end{vmatrix} = i(2-15) - j(1+6) + k(5+4)$$

$$= \langle -13, -7, 9 \rangle$$

$$\langle -26, -14, 18 \rangle = 2\langle -13, -7, 9 \rangle$$

C

7. Find the area of the parallelogram with vertices $P(-5,1,2)$, $Q(-4,3,2)$, $R(-2,1,2)$, and $S(-1,3,2)$.

(A) 2

(B) 3

(C) 4

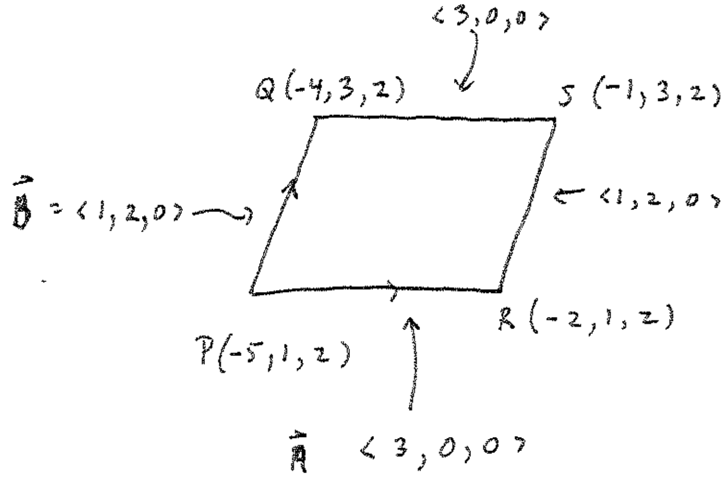
(D) 5

(E) 6

(F) 8

(G) 10

(H) 12



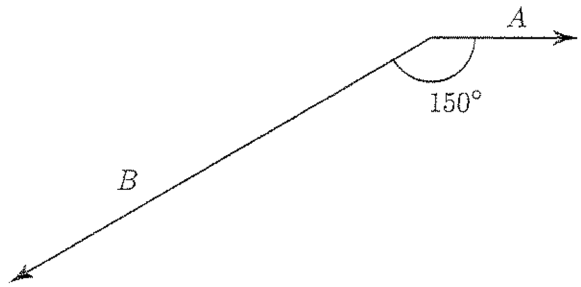
$$A \times B = \begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 1 & 2 & 0 \end{vmatrix} = i(0) - j(0) + k(6)$$

$$= \langle 0, 0, 6 \rangle$$

$\therefore$ area $= |A \times B| = 6$

E

8. Consider the figure below, where $|\mathbf{A}| = 3$, $|\mathbf{B}| = 10$, and the angle between $\mathbf{A}$ and $\mathbf{B}$ is $150°$. Find $|\mathbf{A} \times \mathbf{B}|$ and determine whether $\mathbf{A} \times \mathbf{B}$ points into or out of the page.



(A) $|\mathbf{A} \times \mathbf{B}| = -15$; $\mathbf{A} \times \mathbf{B}$ points into page

(B) $|\mathbf{A} \times \mathbf{B}| = -15$; $\mathbf{A} \times \mathbf{B}$ points out of page

(C) $|\mathbf{A} \times \mathbf{B}| = 15$; $\mathbf{A} \times \mathbf{B}$ points into page

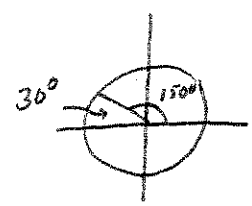
(D) $|\mathbf{A} \times \mathbf{B}| = 15$; $\mathbf{A} \times \mathbf{B}$ points out of page

(E) $|\mathbf{A} \times \mathbf{B}| = -15\sqrt{3}$; $\mathbf{A} \times \mathbf{B}$ points into page

(F) $|\mathbf{A} \times \mathbf{B}| = -15\sqrt{3}$; $\mathbf{A} \times \mathbf{B}$ points out of page

(G) $|\mathbf{A} \times \mathbf{B}| = 15\sqrt{3}$; $\mathbf{A} \times \mathbf{B}$ points into page

(H) $|\mathbf{A} \times \mathbf{B}| = 15\sqrt{3}$; $\mathbf{A} \times \mathbf{B}$ points out of page



$|A \times B| = |A|\,|B| \sin\theta$, $0 \le \theta \le 180°$

$= 30 \sin(150°)$

$= 30 \sin(30°)$

$= 30 \cdot \frac{1}{2}$

$= 15$

C

9. Find the volume of the parallelepiped determined by the vectors

$$\mathrm{a} = \langle 1, 4, 2 \rangle, \mathrm{b} = \langle -1, 1, 3 \rangle, \mathrm{c} = \langle 3, 1, 4 \rangle$$

(A) 15

(B) -15

(C) $\sqrt{150}$

(D) $-\sqrt{150}$

(E) 45

(F) -45

(G) 61

(H) -61

$$\begin{vmatrix} 1 & 4 & 2 \\ -1 & 1 & 3 \\ 3 & 1 & 4 \end{vmatrix} = (4-3) - 4(-4-9) + 2(-1-3)$$

$$= 1 + 52 - 8$$

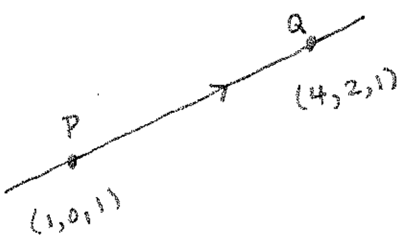
$$= 45$$

E

10. At what point does the line through $(1,0,1)$ and $(4,2,1)$ intersect the plane

$$x+y+z=4\ ?$$

(A) $(\frac{1}{2},\frac{1}{2},3)$
(B) $(-\frac{1}{3},\frac{2}{3},\frac{11}{3})$
(C) $(\frac{3}{4},\frac{5}{4},2)$
(D) $(\frac{11}{5},\frac{4}{5},1)$
(E) $(\frac{20}{7},\frac{8}{7},0)$
(H) $(\frac{8}{7},\frac{20}{7},0)$
(G) $(\frac{8}{9},\frac{1}{9},3)$
(F) $(\frac{22}{13},\frac{4}{13},2)$



$\vec{v} = \vec{PQ} = \langle 3, 2, 0 \rangle$

$P = (1,0,1)$

$\therefore$ parametric eqns:

$$\begin{cases} x = 1+3t \\ y = 2t \\ z = 1 \end{cases}$$

$\therefore \quad x+y+z=4$

$\Rightarrow \quad (1+3t) + (2t) + 1 = 4$

$5t = 2$

$t = 2/5$

$\therefore \quad x = 1 + 3(2/5) = 11/5$

$y = 2(2/5) = \frac{4}{5}$

$z = 1$

$\therefore \quad (\frac{11}{5}, \frac{4}{5}, 1)$

D

11. Let P be the plane which contains the point $(1, 1, 3)$ and is perpendicular to the z-axis. Which of the following sets of equations correctly describes the intersection of P with the sphere of radius 5 centered at the origin? If the plane and sphere are disjoint, write the answer as DNE.

(a) DNE

(A) $x^2 + z^2 = 9$ and $z = 1$

(B) $x^2 + z^2 = 9$ and $z = 3$

(C) $x^2 + y^2 = 9$ and $z = 3$

(D) $y^2 + z^2 = 9$ and $z = 3$

(E) $y^2 + z^2 = 16$ and $z = 3$

(F) $x^2 + y^2 = 16$ and $z = 3$

(G) $x^2 + z^2 = 16$ and $z = 3$

(H) $x^2 + y^2 + z^2 = 25$ and $z = 1$

(J) $x^2 + y^2 + z^2 = 16$ and $z = 3$

P is plane $z = 3$

sphere : $x^2 + y^2 + z^2 = 25$

$$\left.\begin{array}{l} z = 3 \\ x^2 + y^2 + z^2 = 25 \end{array}\right\} \quad \begin{array}{l} x^2 + y^2 + 9 = 25 \\ x^2 + y^2 = 16 \end{array}$$

$\therefore$ $\boxed{\begin{array}{l} x^2 + y^2 = 16 \\ z = 3 \end{array}}$

F

12. Determine whether the planes

$$\sqrt{5}x + 4y - 2z = 1\,, -\sqrt{5}x + 2y + 2z = 2004$$

are parallel, perpendicular or neither. If neither, find the angle between them.

(A) parallel

(B) perpendicular

(C) neither; $cos^{-1}(\frac{1}{3})$

(D) neither; $cos^{-1}(-\frac{1}{3})$

(E) neither; $cos^{-1}(\frac{1}{5\sqrt{13}})$

(F) neither; $cos^{-1}(-\frac{1}{5\sqrt{13}})$

(G) neither; $cos^{-1}(\frac{1}{3\sqrt{5}})$

(H) neither; $cos^{-1}(-\frac{1}{3\sqrt{5}})$

$\vec{n}_1 = \langle \sqrt{5}, 4, -2 \rangle$ $\quad |\vec{n}_1| = \sqrt{5+16+4} = 5$

$\vec{n}_2 = \langle -\sqrt{5}, 2, 2 \rangle$ $\quad |\vec{n}_2| = \sqrt{5+4+4} = \sqrt{13}$

$$\cos\theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1|\,|\vec{n}_2|} = \frac{|-5+8-4|}{5\sqrt{13}} = \frac{1}{5\sqrt{13}}$$

$$\therefore \quad \theta = \cos^{-1}\left(\frac{1}{5\sqrt{13}}\right)$$

E

13. Which of the following could be a vector equation for the line of intersection of the planes

$$x + y + z = 5$$

and

$$x + 4y + 4z = 5$$

(A) $\mathbf{r}(t) = \langle 1+t, 2t, 6t \rangle$

(B) $\mathbf{r}(t) = \langle 3, 3t, -3t \rangle$

(C) $\mathbf{r}(t) = \langle 1+t, 3t, -3t \rangle$

(D) $\mathbf{r}(t) = \langle 5, 2t, 3t \rangle$

(E) $\mathbf{r}(t) = \langle 0, 13t, 1+t \rangle$

(F) $\mathbf{r}(t) = \langle 2+3t, 1+t, 2t \rangle$

(G) $\mathbf{r}(t) = \langle 3+2t, -2t, 1+5t \rangle$

(H) $\mathbf{r}(t) = \langle 5, t, -t \rangle$

$\vec{n}_1 = \langle 1, 1, 1 \rangle$

$\vec{n}_2 = \langle 1, 4, 4 \rangle$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 4 & 4 \end{vmatrix} = i(0) - j(3) + k(3)$$

$$= \langle 0, -3, 3 \rangle$$

$\therefore$ vector in line is scalar multiple of $\langle 0, -1, 1 \rangle$

$\therefore$ only possibilities are B, H

B contains $(3,0,0) \leftarrow$ doesn't satisfy eqn of planes

H contains $(5,0,0) \leftarrow$ does satisfy eqn of planes.

$\therefore$ H

14. Find the domain of the vector function

$$\mathbf{r}(t) = \left(\sqrt{36 - t^2}, e^{-3t}, ln(t+5)\right).$$

(A) $(-5, \infty)$

(B) $[-6, \infty)$

(C) $(-6, 6]$

(D) $[-5, 5]$

(E) $(-5, 5)$

(F) $(-5, 6)$

(G) $(-5, 6]$

(H) $(-5, 9]$

domain $\sqrt{36-t^2} = [-6, 6]$

domain $e^{-3t} = \mathbb{R}$

domain $ln(t+5) = (-5, \infty)$

$\therefore \quad (-5, 6] \qquad$ G

15. If $\mathrm{r}(t) = \langle 2t, 3t^2, 3t^3 \rangle$, find $\mathrm{r}''(0)$.

(A) $\langle 0, 0, 0 \rangle$

(B) $\langle 2, 0, 0 \rangle$

(C) $\langle 0, 6, 18 \rangle$

(D) $\langle 2, 3, 3 \rangle$

(E) $\langle 2, 6, 9 \rangle$

(F) $\langle 0, 6, 0 \rangle$

(G) $\langle 0, 2, 3 \rangle$

(H) $\langle 0, 2, 9 \rangle$

$\vec{r}'(t) = < 2, 6t, 9t^2 >$

$\vec{r}''(t) = < 0, 6, 18t >$

$\vec{r}''(0) = < 0, 6, 0 >$

F

16. Evaluate

$$\int_0^1 (2t\,\mathrm{i} + \frac{4}{1+t^2}\,\mathrm{j} + \frac{5t^4}{1+t^5}\,\mathrm{k})dt$$

(A) $1 + \pi + ln2$

(B) $2 + \pi + ln2$

(C) $2 + \frac{\pi}{4} + ln2$

(D) $1 + \frac{\sqrt{2}}{2} + ln2$

(E) $\langle 1, \pi, ln2 \rangle$

(F) $\langle 2, \pi, ln2 \rangle$

(G) $\langle 2, \frac{\pi}{4}, ln2 \rangle$

(H) $\langle 1, \frac{\sqrt{2}}{2}, ln2 \rangle$

$$\vec{i} \int_0^1 2t\,dt + \vec{j} \int_0^1 \frac{4}{1+t^2}\,dt + \vec{k} \int_0^1 \frac{5t^4}{1+t^5}\,dt$$

$$\vec{i}\left[t^2 \Big|_0^1 \right] + \vec{j}\left[4\tan^{-1} t \right]_0^1 + \vec{k}\left[\ln(1+t^5) \right]_0^1$$

$$= \vec{i} + \vec{j}\left(4 \cdot \frac{\pi}{4}\right) + \vec{k} \ln 2$$

$$= \langle 1, \pi, \ln 2 \rangle$$

E