Extra Limit Problems

February 15, 2013

1. Given $\epsilon > 0$ and
$$f(x,y) = x^2 + y^2$$
find the largest $\delta > 0$ such that $|f(x,y) - 0| < \epsilon$ whenever $0 < \sqrt{x^2 + y^2} < \delta$.

2. Given $\epsilon > 0$ and
$$f(x,y) = \frac{x^2y^2}{x^2 + y^2}$$
find the largest $\delta > 0$ such that $|f(x,y) - 0| < \epsilon$ whenever $0 < \sqrt{x^2 + y^2} < \delta$.

3. Given $\epsilon > 0$ and
$$f(x,y) = \frac{3x^2y^2}{x^2 + y^2}$$
find the largest $\delta > 0$ such that $|f(x,y) - 0| < \epsilon$ whenever $0 < \sqrt{x^2 + y^2} < \delta$.

4. Given $\epsilon > 0$ and
$$f(x,y) = \frac{y^2}{4 + cosx}$$
find the largest $\delta > 0$ such that $|f(x,y) - 0| < \epsilon$ whenever $0 < \sqrt{x^2 + y^2} < \delta$.

DO NOT TURN THE PAGE UNTIL YOU WANT TO SEE ANSWERS.

ANSWERS:

(Notice that you are showing that $lim_{(x,y)\to(0,0}f(x,y) = 0$ in each case.)

1. If $\sqrt{x^2+y^2} < \delta$, then

$$|f(x,y) - 0| = |x^2 + y^2| < \delta^2$$

   So choosing δ so that $\delta^2 = \epsilon$ works; namely, choose $\delta = \sqrt{\epsilon}$.

2. If $\sqrt{x^2+y^2} < \delta$, then

$$|f(x,y) - 0| = |\frac{x^2y^2}{x^2+y^2}| \leq \frac{(x^2+y^2)(x^2+y^2)}{x^2+y^2} = x^2 + y^2 < \delta^2$$

   So choosing δ so that $\delta^2 = \epsilon$ works; namely, choose $\delta = \sqrt{\epsilon}$.

3. If $\sqrt{x^2+y^2} < \delta$, then

$$|f(x,y) - 0| = |\frac{3x^2y^2}{x^2+y^2}| \leq \frac{3(x^2+y^2)(x^2+y^2)}{x^2+y^2} = 3(x^2 + y^2) < 3\delta^2$$

   So choosing δ so that $3\delta^2 = \epsilon$ works; namely, choose $\delta = \sqrt{\epsilon/3}$.

4. If $\sqrt{x^2+y^2} < \delta$, then

$$|f(x,y) - 0| = |\frac{y^2}{4 + cos\, x}| \leq \frac{y^2}{3} \leq \frac{x^2+y^2}{3} < \delta^2/3$$

   So choosing δ so that $\delta^2/3 = \epsilon$ works; namely, choose $\delta = \sqrt{3\epsilon}$.