Math 132
Final Examination – May 4, 2012

6 multiple choice, 4 long answer. 100 points.

General Instructions: Please answer the following, without use of calculators. You may refer to up to four 3x5 cards, but no other notes. Part I of the exam is multiple choice, while Part II is long answer.

Part I Instructions: If you do not have a pencil to fill out your answer card, please ask to borrow one from your proctor. Write your Student ID number on the six blank lines on the top of your answer card, and shade in the corresponding bubbles to the right of each digit.
Fill in the bubble corresponding to each of the following 6 questions. Each is worth 4 points. On Part I, no partial credit will be given.

1. The Taylor series for $f(x) = \dfrac{1}{1-x^3}$ (around 0) is

(a) $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k!}$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{x^{3k}}{k!}$

(c) $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{(3k)!}$

(d) $\displaystyle\sum_{k=0}^{\infty} x^{3k}$

(e) $\displaystyle\sum_{k=3}^{\infty} x^k$

(f) $\displaystyle\sum_{k=3}^{\infty} \frac{x^k}{k!}$

(g) $\displaystyle\sum_{k=3}^{\infty} x^{3k}$

(h) None of the above.

2. Evaluate $\sum_{i=2}^{\infty} 16 \cdot \left(-\frac{3}{4}\right)^{i}$.

(a) 0

(b) $\frac{4}{7}$

(c) 1

(d) 2

(e) 3

(f) 4

(g) 5

(h) $\frac{36}{7}$

(i) Does not converge – oscillates.

(j) Does not converge – diverges to ∞.

3. Evaluate $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{8i^3}{n^4}$ by recognizing it as a Riemann sum.

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) $\frac{3}{2}$

(e) 2

(f) $\frac{5}{2}$

(g) 3

(h) $\frac{7}{2}$

(i) 4

(j) $\frac{9}{2}$

(k) Diverges to ∞

4. Which of the following series can the Alternating Series test be used on?

I. $\displaystyle\sum_{i=0}^{\infty} \frac{(-1)^i}{i^2+1}$

II. $\displaystyle\sum_{i=0}^{\infty} \frac{(-1)^i}{i\cos^2 i}$

III. $\displaystyle\sum_{i=1}^{\infty} \frac{(-1)^i}{ie^i}$

(a) None of them.

(b) I only.

(c) II only.

(d) III only.

(e) I and II only.

(f) I and III only.

(g) II and III only.

(h) All of I, II, and III.

5. Evaluate $\lim_{i\to\infty} \dfrac{2^{i+1}+i^2}{2^i+3i^3}$.

(a) 0
(b) $\frac{1}{3}$
(c) $\frac{2}{5}$
(d) $\frac{1}{2}$
(e) $\frac{2}{3}$
(f) 1
(g) $\frac{4}{3}$
(h) $\frac{3}{2}$
(i) 2
(j) 3
(k) Diverges to ∞.

6. Evaluate $\int_0^{\pi} x\sin(\frac{x}{2})\,dx$.

(a) -2π
(b) -4
(c) $-\pi$
(d) -2
(e) 0
(f) 1
(g) 2
(h) π
(i) 4
(j) 2π
(k) Diverges.

Name: ____________________ Id #: __________ Math 132

Part II Instructions: Answer the following on the exam sheet, showing all your work. Correct answers without correct supporting work may not receive full credit. You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Please put your name and student id number on **each page** of Part II now.

1. Calculations

   (a) (7 points) Find all solutions to the differential equation $y' = \dfrac{y}{1-x}$ (assume $x < 1$).

   (b) (6 points) Find an upper bound for $|2e^{-x} + 3e^{x} + 4\sin x + 5\cos x|$ on the interval $(-1, 3)$.

   (c) (6 points) Using a power series, find $f^{(100)}(0)$ and $f^{(101)}(0)$ for $f(x) = e^{x^2}$.

2. Integrals

(a) (5 points) Set up an integral for the area of the surface obtained by rotating the curve $y = e^{-2x}$ around the x-axis, for x between 0 and ∞.
(You need not evaluate the integral in question.)

(b) (6 points) Show that the improper integral $\displaystyle\int_0^\infty \frac{1}{1+x^4}\,dx$ converges.

(c) (8 points) Find the volume of the solid obtained by rotating the region below the curve $y = \dfrac{1}{3x+1}$ about the x-axis for $0 \leq x < \infty$.

Name: ______________________________ Id #: __________ Math 132

3. Series and power series. Use the back if you need additional space.

   (a) (7 points) Using partial fractions, find a power series representation for $\dfrac{4}{(x-1)(x-3)}$.

   (b) (8 points) Find the radius and interval of convergence for the power series $\displaystyle\sum_{i=0}^{\infty} \frac{2^i}{i^2+2} x^i$.

   (c) (6 points) Does the series $\displaystyle\sum_{i=0}^{\infty} \frac{(-1)^i}{i^{3/2}-1}$ converge absolutely, converge conditionally, or diverge?

Name: ______________________ Id #: __________ Math 132

4. Integrating $\cos x^2$.

   (a) (1 point) We have discussed that $\cos x^2$ has no closed form antiderivative. In 1-2 sentences, explain what this means.

   (b) (6 points) Using the Fundamental Theorem of Calculus and an appropriate definite integral, give an antiderivative of $\cos x^2$.

   (c) (6 points) Using a Taylor series expansion, give a power series representation of an antiderivative of $\cos x^2$.

   (d) (2 points) Using part (c), find a series representing $\int_0^1 \cos x^2 \, dx$.