Ma 494 — Theoretical Statistics

Problem Set #2 — Due February 12, 2009

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1. Problem 5.2.20 page 363. (Note: The second part of the problem asks you to compare your answer to your answer for the maximum likelihood estimator for the same problem in Problem 5.2.12 in HW#1, so keep your returned homework.)

2. Problem 5.4.6 page 387.

3. Problem 5.4.14 page 388.

4. (Like Problem 5.5.2 page 397.) Let $X_1, X_2, \ldots, X_n$ be a random sample from $f(x, \theta) = (1/\theta)e^{-x/\theta}$ for $x > 0$. Is $\overline{X} = (1/n)\sum_{k=1}^{n} X_k$ an efficient estimator of θ, in the sense of the definition on page 396?

5. Let $X_1, X_2, \ldots, X_n$ be a random sample from $f(x, \theta) = (1/\theta)e^{-x/\theta}$ for $x > 0$. Prove that

$$T \;=\; \frac{1}{\Gamma\big(\frac{n+1}{n}\big)^n}\,(X_1 X_2 \ldots X_n)^{1/n}$$

is an unbiased estimator of θ.