Ma 494 — Theoretical Statistics

Problem Set #4 — Due March 3, 2010

Prof. Sawyer — Washington University

NOTE: 5 problems on 2 pages.

1. (Somewhat like Problem 5.7.1 page 409) Let $X_1, X_2, \ldots, X_n$ be independent and normally distributed with mean $\mu = 3$ and standard deviation $\sigma_0 = 1$. What is the smallest value of the integer n such that $P\big(\overline{X}_n \in (2.5, 3.5)\big) \geq 0.99$ for $\overline{X}_n = (1/n)\sum_{k=1}^{n} X_k$? (*Hint*: See Table A.1 in the back of the book.)

2. Let $X_1, \ldots, X_7$ be independent normal with unknown mean μ and standard deviation $\sigma = 1.2$. Suppose that $\overline{X} = 0.80$.

(i) Find a symmetric 95% confidence interval for the unknown μ.

(ii) Find a lower one-sided 95% confidence interval for μ. That is, find a random interval $\big(A(X), \infty\big)$ with $A(x) = \overline{X} - c_n$ such that, with probability 0.95, the true $\mu \in \big(A(X), \infty\big)$.

Note that $\mu = 0$ is included on one of the two intervals but not the other. Conclude that μ is positive with 95% certainly, but that, from the other confidence interval, with 95% certainty μ cannot be distinguished from zero.

3. (Problem 5.3.8 on page 377) Out of a sample of 220 tuna salads marketed by retail and wholesale food outlets in New York City, a total of 179 were found to be unsatisfactory for health reasons. Find an approximate 90% confidence interval for the true proportion p of contaminated tuna salads marketed in New York City. (*Warning*: Note 90%, not 95%.)

4. (Problem 5.3.14 on page 378) Assume that, in one of the congressional elections in 1994, the challenger George Nethercutt had a lead of 2174 votes (50.6%) over the reigning Speaker of the House, Tom Foley, with most votes counted but 14,000 absentee ballots remaining to be counted. (Absentee ballots are written ballots from voters who were unable to vote in person.) Let p be the probability that an absentee ballot favors Foley with $1 - p$ for Nethercutt. How small can p be and still allow Foley a 20% chance of winning the election?

(*Hints*: $p > 1/2$, since otherwise Nethercutt would likely win the absentee ballots as well. Use the conservative approximation $X/n \approx p + (1/\sqrt{4n})Z$ where X is the number of successes in n Bernoulli trials and Z is a standard normal random variable. (See page 373 in the text.) Note that $P(Z > 0.84) = 0.20$ by Appendix A.1.)

(*Remark*: Tom Foley eventually lost this relatively famous election, but this is not a hint since, by reigning political wisdom at the time, the majority of absentee voters vote Republican. This would favor George Nethercutt.)

5. (Like Problem 5.3.24 on page 379) Suppose that p is to be estimated by the maximum-likelihood estimator X/n. What is the smallest value of n such that X/n has a 99% or greater probability of being within 0.05 of the true value of p? (*Hint*: Use the conservative approximation $X/n \approx p+(1/\sqrt{4n})Z$ mentioned in the previous problem.)