Ma 320 Test 2 – Section 1

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Calculators and a single $8^{1}/_{2}$ by 11 cribsheet can be used. All problems are 15 points unless indicated otherwise.

1. A mail survey of 3440 people was conducted. In the survey, 1376 people reacted positively to a particular TV commercial and the rest did not. Find a 95% confidence interval for the percentage of the general population that would react positively to that TV commercial. (*Hint*: You may need to use an approximation.)

Answer: The formula for a 95% normal confidence interval is $\widehat{p} \pm 1.960\sqrt{\widehat{p}\widehat{q}/n}$. Here $\widehat{p} = 0.4$ and $\widehat{q} = 1 - \widehat{p} = 0.6$. The confidence interval is $0.4 \pm 1.960\sqrt{.4(.6)/2440} = (0.3837,\ 0.4164)$.

2. The average wage in an industry for a particular class of workers is \$13.20 per hour. The standard deviation is \$2.50 per hour. At a particular company that is involved in litigation, the average wage for 40 workers in this category is \$12.20 per hour. Are the wages in this company significantly low at a level of significance $\alpha = 0.10$? What is the P-value? Assume that the hourly wages are normally distributed, and that the industry-wide standard deviation for wages also applies in this company.

Answer: Here $H_0 : \mu = 13.20$ and $H_1 : \mu < 13.20$. The population standard deviation $\sigma = 2.50$ is given. The level of confidence is $\alpha = 0.10$. The observed test statistic is $T = (12.20 - 13.20)/(2.50/\sqrt{40}) = -2.5298$. T has a standard normal distribution given H_0. Here $z(\alpha) = z(0.10) = -1.2816$, so we reject H_0 and conclude that the wages are significantly lower. The P-value is $P = P(T \le -2.5298) = 0.0057$.

3. (i) A company is trying to decide whether to purchase machines of design A or of design B. Eight (8) technicians are used in a comparison between the two types of machines. Each technician carries out the same task on both types of machines. Which type of machine the technician tried out first (design A or design B) was chosen randomly for each technician. The times taken by the technicians are listed below. Assume that the values recorded are normally distributed. Is there a difference in mean completion times, using a level of significance of $\alpha = 0.05$? What is the P-value?

Technician	Machine A	Machine B
1	73	65
2	53	47
3	72	73
4	73	65
5	66	70
6	66	59
7	76	72
8	79	67

(ii) What test statistic did you use? What is its distribution under hypothesis H_0?

Answers: (i) This is a paired design, so that the basic data are the eight paired differences $\{D_i\} = \{8, 6, -1, 8, -4, 7, 4, 12\}$. Here $H_0 : \mu_D = 0$ (or $\mu_A = \mu_B$) and $H_1 : \mu_D \neq 0$ (or $\mu_A \neq \mu_B$). The level of significance is $\alpha = 0.05$. By calculation $\overline{D} = 5$ and $s_D = 5.21$. The observed test statistic $T_D = \overline{D}/(s_D/\sqrt{8}) = 2.7144$. T_D has a Student's-t distribution with 7 degrees of freedom given H_0. Here $t(7, 0.975) = 2.3646$ so we reject H_0 at $\alpha = 0.05$. The P-value is $P = P(|t(7)| \geq 2.7144) = 2P(t(7) \geq 2.7144) = 0.0300$.

(ii) The test statistic I used is $T_D = \overline{D}/(s_D/\sqrt{n})$. T_D has a Student's-t distribution with $n - 1$ degrees of freedom given H_0.

4. (i) A company advertises that repeated measurements of the same source by one of its machines will have a standard deviation of 2.0 or less. A technician buys one of these machines, repeats the same measurement 16 times, and finds that the sample standard deviation is 2.87. Is this evidence that the true standard deviation is greater than 2.0 at level of significance 0.05? What is the P-value?

(ii) What test statistic did you use? What is its distribution under hypothesis H_0?

Answers: (i) Here $H_0 : \sigma = 2.0$ and $H_1 : \sigma > 2.0$. The observed test statistic $T = 15 \times 2.87^2/2.0^2 = 30.888$. T has a chi-square distribution with 15 degrees of freedom given H_0. Here $\chi^2(15, 0.95) = 25.00$ so that we reject H_0 and conclude that the standard deviation is greater than 2.0 at level of significance $\alpha = 0.05$. The P-value is $P = P(\chi^2(15) \geq 30.888) = 0.0091$.

(ii) The test statistic I used is $T = (n - 1)s^2/\sigma^2$. T has a chi-square distribution with $n - 1$ degrees of freedom given H_0.

5. The lifetimes of flood lights at a particular location can be modeled as having an exponential distribution with a mean of 10 days. Find the probability that three or more of the next ten flood lights will last 22 days or longer. (*Hint*: This problem involves both an exponential and a binomial distribution. Recall that the *mean* of an exponential is one over its *rate*.)

Answer: The lifetime L for one flood light satisfies $P(L > t) = \exp(-t/\mu)$ so that $P(L > 22) = \exp(-22/10) = 0.1108$. Now find $P(T \geq 3)$ where T has a binomial distribution with $p = 0.1108$ and $n = 10$. Using a TI-83 calculator, $P(T \leq 2) =$ binomcdf(10,0.1108,2) $= 0.9100$ so that $P(T \geq 3) = 1 - P(T \leq 2) = 0.0900$ since T takes on integer values only.

6. Essay (5 points) Explain the following terms: P-value, level of significance, and power. Given an example to clarify your statements. What is the relationship between the P-value and the level of significance?

7. (Extra Credit) Six different species of parrots are known to live in both inhabited and uninhabited forests in a tropical country. It is thought that parrots living in inhabited areas might tend to be larger. The average weights of parrots of these six species in the two types of forest are:

Parrot Species	Inhabited Forest	Uninhabited Forest
1	168	148
2	85	88
3	66	73
4	67	68
5	62	53
6	75	69

Is there is significant different in the weights of these parrots between inhabited and uninhabited areas? Carry out a Wilcoxon Signed Rank test to find out. What is the sum of the signed ranks? What is the (two-sided) P-value?

Answer: This is a paired design, so that the basic data are the six paired differences $\{D_i\} = \{20, -3, -7, -1, 9, 6\}$. Here $H_0 : \mu_D = 0$ (or $\mu_A = \mu_B$) and $H_1 : \mu_D \neq 0$ (or $\mu_A \neq \mu_B$), which we test using the signed ranks of the D_i instead of the D_i themselves. The level of significance is $\alpha = 0.05$. (This is understood if the level of significance isn't stated explicitly.)

The absolute values are $\{20, 3, 7, 1, 9, 6\}$ and the corresponding ranks are $\{6, 2, 4, 1, 5, 3\}$. The signed ranks are the ranks with the original signs $\{R_i\} = \{6, -2, -4, -1, 5, 3\}$. By calculation, $\overline{R} = 1.167$ and $s_R = 4.07$. The observed test statistic for the signed ranks is $T_R = 1.167/(4.07/\sqrt{6}) = 0.702$. T_R is assumed to have a Student's-t distribution with 5 degrees of freedom given H_0. Here $t(5, 0.975) = 2.5702$, so that we accept H_0 and conclude that there is no evidence for a difference in parrot weights. The P-value is $P = 2P(t(5) \geq 0.702) = 0.5140$.