MATH 544: TOPICS IN GEOMETRY AND MANIFOLD THEORY
FLOWS AND THE GEOMETRY OF 3-MANIFOLDS

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The goal of the class is to understand the relationship between the geometry and topology of a space and the kinds of dynamical systems that it supports, with a focus on low dimensions (i.e. 1, 2, 3).

Possible topics include:

(1) **Overview of 3-Manifolds:** Triangulations, Dehn surgery, surface bundles, sutured manifold hierarchies. The Poincare conjecture, JSJ decompositions, geometrization.

(2) **Hyperbolic Geometry**: Hyperbolic geometry. Thurston's hyperbolization for surface bundles, Cannon-Thurston maps. Cannon conjecture. Thurston norm.

(3) **Fibrations and Foliations.**

(4) **Homology Directions:** A flow on a manifold M has a corresponding set of *homology directions*, which one can think of as a subset of the unit sphere in the vector space $H_1(M;\mathbb{R})$ (together with the origin, thought of as the trivial direction). This can be used to understand when a flow is transverse to a fibration or foliation. In particular, Fried's Theorem says that a flow is transverse to a codimension-1 fibration if and only if its homology directions are contained in a hemisphere.

(5) **Hyperbolicity in dynamics**: Quasigeodesic flows, Anosov flows, pseudo-Anosov flows.

Students should ideally have a working knowledge of fundamental groups, covering spaces, homology, and cohomology. No prior knowledge of 3-manifolds will be assumed.

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