MATH 544: TOPICS IN GEOMETRY AND MANIFOLD THEORY
FLOWS AND THE GEOMETRY OF 3-MANIFOLDS

STEVEN FRANKEL

The goal of the class is to understand the relationship between the geometry and
topology of a space and the kinds of dynamical systems that it supports, with a
focus on low dimensions (i.e. 1, 2, 3).
Possible topics include:

1. **Overview of 3-Manifolds**: Triangulations, Dehn surgery, surface bundles, sutured manifold hierarchies. The Poincare conjecture, JSJ decompositions, geometrization.


3. **Fibrations and Foliations**.

4. **Homology Directions**: A flow on a manifold $M$ has a corresponding set
   of homology directions, which one can think of as a subset of the unit sphere
   in the vector space $H_1(M; \mathbb{R})$ (together with the origin, thought of as the
   trivial direction). This can be used to understand when a flow is transverse
to a fibration or foliation. In particular, Fried’s Theorem says that a flow is
transverse to a codimension-1 fibration if and only if its homology directions
are contained in a hemisphere.

5. **Hyperbolicity in dynamics**: Quasigeodesic flows, Anosov flows, pseudo-
   Anosov flows.

Students should ideally have a working knowledge of fundamental groups, cov-
ering spaces, homology, and cohomology. No prior knowledge of 3-manifolds will
be assumed.

Department of Mathematics, Washington University in St. Louis, St. Louis, MO
E-mail address: steven.frankel@wustl.edu