Math 131E
Krantz

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SOLUTIONS TO PRACTICE EXAM FOR FIRST MIDTERM

1 a) We must have $1-x^2 \geq 0$ or $x^2 \leq 1$ or $-1 \leq x \leq 1$. So domain is $[-1, 1]$.

b) We cannot divide by 0 so $x^2 \neq 1$ hence $x \neq \pm 1$. Thus domain is $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$.

c) tan is undefined when $\cos x = 0$ or x is an odd multiple of $\pi/2$. So

domain $= \{x \in \mathbb{R} : x$ is <u>not</u> an odd multiple of $\pi/2\}$

2. $$\ln(x^2-2) = 4$$
$$e^{\ln(x^2-2)} = e^4$$
$$x^2 - 2 = e^4$$
$$x^2 = 2 + e^4$$
$$x = \pm\sqrt{2+e^4}$$

3. Let $y = x^2 + x$. So $x^2 + x - y = 0$. By quadratic formula,
$$x = \frac{-1 \pm \sqrt{1+4y}}{2}.$$

By the restriction on the domain we choose +. So the inverse is
$$f^{-1}(x) = \frac{-1 + \sqrt{1+4x}}{2}.$$

4. $f \circ g \circ h(x) = f(g(h(x))) = f(g(\sin x))$
$= f(1-\sin^2 x) = e^{1-\sin^2 x}$.

5. $$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[(x+h)-(x+h)^2] - [x - x^2]}{h}$$

$$= \lim_{h \to 0} \frac{[(\cancel{x}+h) - (\cancel{x^2} + 2xh + h^2)] - [\cancel{x} - \cancel{x^2}]}{h}$$

$$= \lim_{h \to 0} \frac{h - 2xh - h^2}{h}$$

$$= \lim_{h \to 0} (1 - 2x - h)$$

$$= 1 - 2x.$$

6. $g'(x) = 2x + 1$. So $g'(1) = 2 \cdot 1 + 1 = 3$. Thus ~~tangent line is~~

$$\frac{y-2}{x-1} = m = 3$$

$$y - 2 = 3x - 3$$

$$y = 3x - 1$$

7. $f'(x) = \cos(\sqrt{x+x^2}) \cdot \frac{1}{2}(x+x^2)^{-1/2} \cdot (1+2x)$.

8. $h'(x) = 2x \cdot \cos x + x^2 \cdot (-\sin x)$

9. $$k'(x) = \frac{(\sin x) \cdot (2x+1) - (x^2+x) \cdot \cos x}{\sin^2 x}$$

10. $m'(x) = [\cos(x^3 - x^2)] \cdot (3x^2 - 2x).$