Math 131E
Krantz

Fall, 2021
October 28, 2021

FIRST MIDTERM

General Instructions: Read the statement of each problem carefully. If you want full credit on a problem then you must show your work. If you only write the answer then you will *not* receive full credit.

Be sure to ask questions if anything is unclear. This exam has 10 questions and is worth 100 points. You will have one hour to take this exam.

If you need extra space then write on the back of the page.

(9 points) **1.** Write the domain of each function

(a) $f(x) = \sqrt{8 - 2x^2}$ Need $8 - 2x^2 \geq 0 \Rightarrow x^2 \leq 4$
so $-2 \leq x \leq 2$.

(b) $g(x) = 1/(1 - x^3)$ Cannot have $x^3 = 1$, So $x \neq 1$.
Domain $= \{x \in \mathbb{R} : x \neq 1\}$.

(c) $h(x) = \cot x$ $\cot x = \frac{\cos x}{\sin x}$. Cannot have $\sin x = 0$.
So $x \neq n\pi$, n an integer.
domain $= \{x \in \mathbb{R} : x \neq n\pi, n \in \mathbb{Z}\}$.

(10 points) **2.** Solve the equation

$$\ln(x^2 + 3) = 4$$

for x.

$$e^{\ln(x^2+3)} = e^4$$

$$x^2 + 3 = e^4$$

$$x^2 = e^4 - 3$$

$$x = \pm\sqrt{e^4 - 3}$$

(12 points) **3.** Let $f(x) = x^2 - x$ for $x \geq 1$. Calculate f^{-1}.

$$y = x^2 - x$$

$$x^2 - x - y = 0$$

$$x = \frac{1 \pm \sqrt{1+4y}}{2}$$

Since $x \geq 1$ we take the solution with $+$.

$$x = \frac{1 + \sqrt{1+4y}}{2}$$

The inverse is $f^{-1}(x) = \dfrac{1 + \sqrt{1+4x}}{2}$.

(10 points) **4.** Let $h(x) = e^x$, $g(x) = 1 + x^2$, $f(x) = \sin x$. Write the function $m(x) = f \circ g \circ h(x)$ explicitly.

$$f \circ g \circ h(x) = f(g(h(x))) = \sin(g(h(x)))$$
$$= \sin(1 + h^2(x))$$
$$= \sin(1 + e^{2x}).$$

(10 points) **5.** Use a limit to calculate the derivative of the function $f(x) = x + x^2$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{[(x+h) + (x+h)^2] - [x + x^2]}{h}$$
$$= \lim_{h \to 0} \frac{x + h + x^2 + 2xh + h^2 - x - x^2}{h}$$
$$= \lim_{h \to 0} \frac{h + 2xh + h^2}{h}$$
$$= \lim_{h \to 0} 1 + 2x + h$$
$$= 1 + 2x$$

(10 points) **6.** Find the tangent line to the curve $g(x) = x^2 - x$ at the point $(1, 0)$.

$g'(x) = 2x - 1$. $g'(1) = 2 \cdot 1 - 1 = 1$.

Tangent line is $\frac{y-0}{x-1} = 1 \Rightarrow y = x - 1$

(10 points) **7.** Calculate the derivative of $f(x) = \cos(\sqrt{x - x^2})$.

$f'(x) = -\sin(\sqrt{x - x^2}) \cdot \frac{1}{2}(x - x^2)^{-1/2} \cdot (1 - 2x)$.

(10 points) **8.** Calculate the derivative of $h(x) = x^3 \cdot \sin x$.

$$h'(x) = 3x^2 \cdot \sin x + x^3 \cdot \cos x \; .$$

(9 points) **9.** Calculate the derivative of

$$k(x) = \frac{x^2 - x}{\cos x}.$$

$$k'(x) = \frac{\cos x \cdot (2x - 1) - (x^2 - x) \cdot (-\sin x)}{\cos^2 x} .$$

(10 points) **10.** Calculate the derivative of $m(x) = \sin(x^3 + x^2)$.

$$m'(x) = [\cos(x^3+x^2)] \cdot [3x^2 + 2x].$$