Math 131E Krantz

Fall, 2021

SOLUTIONS TO PRACTICE MIDTERM 2

1. Let $u = \ln x$

$$\frac{du}{dx} = \frac{1}{x} \qquad x = 3 \Rightarrow u = \ln 3$$

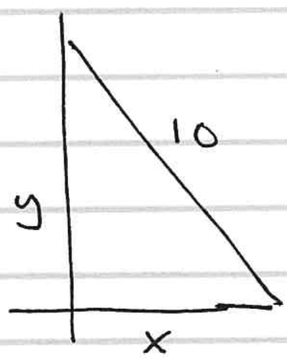
$$x = 5 \Rightarrow u = \ln 5$$

$$du = \frac{1}{x}\,dx$$

So $$\int_3^5 (\ln x)^2 \cdot \frac{1}{x}\,dx = \int_{\ln 3}^{\ln 5} u^2\,du = \frac{u^3}{3}\Big]_{\ln 3}^{\ln 5}$$

$$= \frac{(\ln 5)^3}{3} - \frac{(\ln 3)^3}{3}.$$

2.



$$y = \sqrt{100 - x^2} = (100 - x^2)^{1/2}$$

$$\frac{dy}{dt} = \frac{1}{2}(100 - x^2)^{-1/2} \cdot (-2x) \cdot \frac{dx}{dt}$$

$$= \frac{1}{2}(100 - 6^2)^{-1/2} \cdot (-2 \cdot 6) \cdot 4$$

$$= \frac{1}{2}(64)^{-1/2} \cdot (-12) \cdot 4$$

$$= \frac{1}{2} \cdot \frac{1}{8} \cdot (-48) = \frac{-48}{16} = -3 \text{ ft./sec.}$$

3. Let $u = \cos x$

$$\frac{du}{dx} = -\sin x$$

$$du = -\sin x\,dx$$

So $\int (\cos x)^3 \cdot \sin x \, dx = -\int (\cos x)^3 \cdot (-\sin x)\, dx$

$$= -\int u^3 \, du = -\frac{u^4}{4} + C$$

$$= -\frac{(\cos x)^4}{4} + C$$

4. $\frac{ds}{dt} = 2t - 6$. Clearly $\frac{ds}{dt} < 0$ when $t < 3$

and $\frac{ds}{dt} > 0$ when $t > 3$.

On $[1,3]$,

$$s(3) - s(1) = \int_1^3 s'(t)\,dt = \int_1^3 2t - 6 \, dt$$

$$= t^2 - 6t \Big]_1^3 = (9-18) - (1-6)$$

$$= -4.$$

On $[3,6]$,

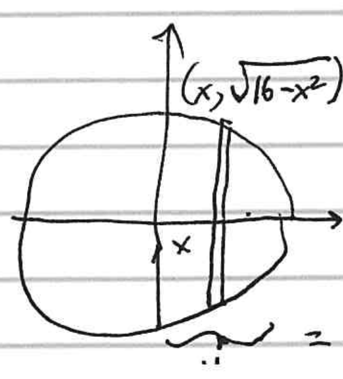
$$s(6) - s(3) = \int_3^6 s'(t)\,dt = \int_3^6 2t - 6 \, dt$$

$$= t^2 - 6t \Big]_3^6 = (36-36) - (9-18) = 9.$$

Total distance traveled is

$$-(-4) + 9 = 13.$$

5.



$$\text{volume} = \int_{-4}^{4} \pi \left(\sqrt{16-x^2}\right)^2 dx$$

$$= \pi \int_{-4}^{4} 16 - x^2 \, dx = \pi\left(16x - \frac{x^3}{3}\right)\Big]_{-4}^{4}$$

$$= \pi\left[\left(64 - \frac{64}{3}\right) - \left(-64 + \frac{64}{3}\right)\right] = \pi\left(128 - \frac{128}{3}\right) = \pi \cdot \frac{256}{3}.$$

6. $xy^2 + y^2 = 2x^2 y$

$$y^2 + x\,2y\frac{dy}{dx} + 2y\frac{dy}{dx} = 4xy + 2x^2\frac{dy}{dx}$$

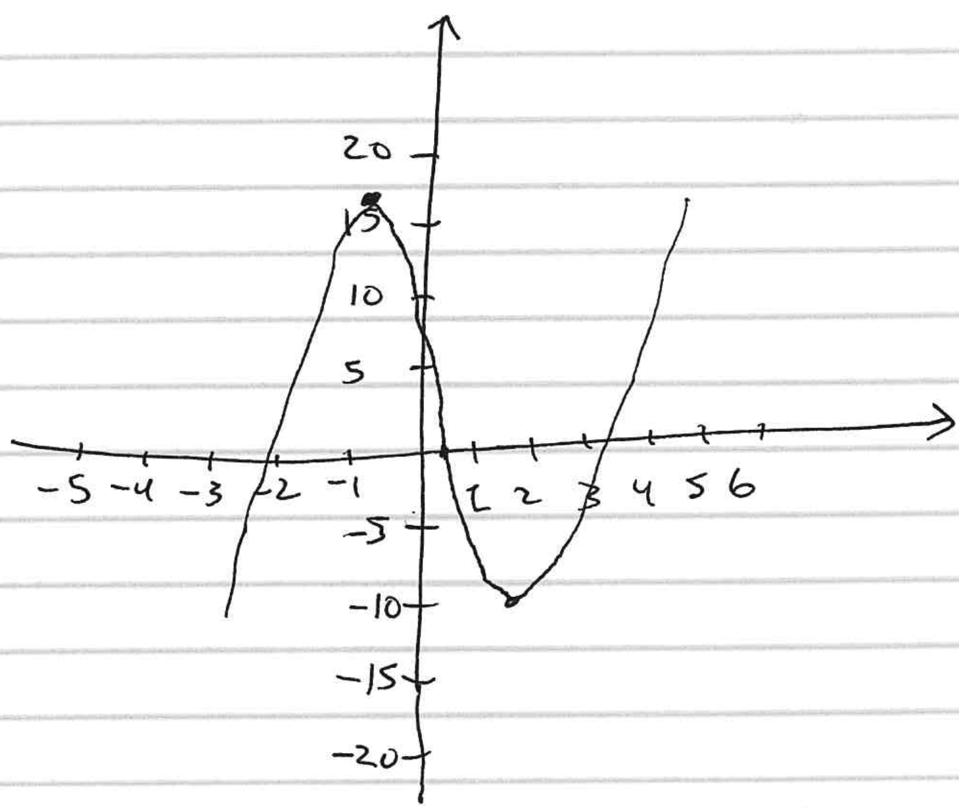
$$\frac{dy}{dx}\left(2xy + 2y - 2x^2\right) = 4xy - y^2$$

$$\frac{dy}{dx} = \frac{4xy - y^2}{2xy + 2y - 2x^2}.$$

At $(1,1)$, we have

$$\frac{dy}{dx} = \frac{4-1}{2+2-2} = \frac{3}{2}.$$

7.



$$\frac{dy}{dx} = 6x^2 - 6x - 12$$
$$= 6(x^2 - x - 2) = 6(x+1)(x-2).$$

So critical points are $x = -1$, $x = 2$.

$$\frac{d^2y}{dx^2} = 12x - 6$$

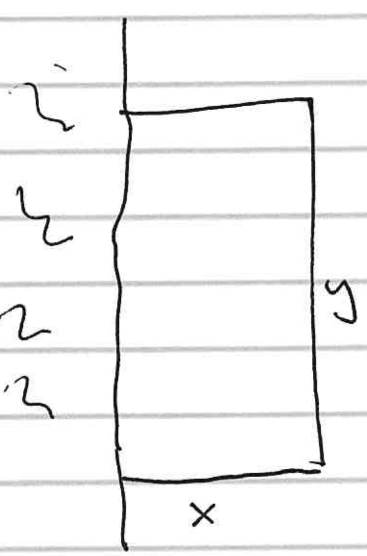
Hence -1 is a local maximum,
2 is a local minimum.

The curve is concave down for $x < \frac{1}{2}$,
concave up for $x > \frac{1}{2}$. Finally,

$x = -1 \Rightarrow y = -2 - 3 + 12 + 10 = 17$

$x = 2 \Rightarrow y = 16 - 12 - 24 + 10 = -10.$

8.



$$y + 2x = 2400$$
$$y = 2400 - 2x$$
$$A = x \cdot y$$
$$= x \cdot (2400 - 2x)$$
$$= 2400x - 2x^2$$
$$A' = 2400 - 4x = 0$$
$$x = 600$$
$$y = 1200.$$

9. $x^2 - 1 = -2x^2 + 8$

$3x^2 = 9$

$x^2 = 3$

$x = \pm\sqrt{3}$

$$A = \int_{-\sqrt{3}}^{\sqrt{3}} (-2x^2+8) - (x^2 - 1)\,dx$$

$$= \int_{-\sqrt{3}}^{\sqrt{3}} -3x^2 + 9\,dx = -x^3 + 9x \Big]_{-\sqrt{3}}^{\sqrt{3}}$$

$$= (-3\sqrt{3} + 9\sqrt{3}) - (+3\sqrt{3} - 9\sqrt{3})$$

$$= -6\sqrt{3} + 18\sqrt{3} = 12\sqrt{3}.$$

10. $E(t) = C \cdot e^{kt}$

$10 = E(0) = C \cdot e^{k \cdot 0} = C$

So $E(t) = 10 \cdot e^{kt}$

$5 = E(20) = 10 \cdot e^{k \cdot 20}$

$e^{k \cdot 20} = \frac{1}{2} \Rightarrow k \cdot 20 = \ln\left(\frac{1}{2}\right) = -\ln 2$

$k = -\frac{1}{20} \ln 2.$

So $E(t) = 10\, e^{-\left(\frac{1}{20}\ln 2\right)t}$

$E(30) = 10^{-\frac{30}{20}\ln 2} = 10^{-\frac{3}{2}\ln 2}.$