PRACTICE EXAM FOR THIRD MIDTERM

1. What is the Fourier series of

\[ f(x) = \begin{cases} 
-1 & \text{if } -\pi \leq x < 0 \\ 
1 & \text{if } 0 \leq x \leq \pi.
\end{cases} \]

2. The Fourier series of the function \( g(x) = \sin x \) on the interval \([\pi/2, \pi/2]\) is what?

3. Find the sine series for the function

\[ h(x) \equiv 1 \]

on the interval \([0, \pi]\).

4. The reason that we do Fourier analysis with functions \( \sin j\pi x/L \) and \( \cos j\pi x/L \) on the interval \([-L, L]\) (rather than some other interval) is what?

5. According to Dirichlet’s theorem, the Fourier series of the function

\[ f(x) = \begin{cases} 
2 & \text{if } -\pi \leq x < 0 \\ 
-2 & \text{if } 0 \leq x \leq \pi
\end{cases} \]

at the point \( c = 0 \) converges to what value?

6. The eigenvalues \( \lambda_n \) and the eigenfunctions \( y_n \) for the equation \( y'' + \lambda y = 0 \) for the boundary conditions \( y(0) = 0, \ y(2\pi) = 0 \) are what?

7. An insulated rod has initial temperature 5°C uniformly on the entire rod. The ends of the rod are held at temperature 0°C. The solution of the heat equation for this rod is then what?
8. The solution of the Dirichlet problem on the unit disc with boundary data function

\[ f(\theta) = \begin{cases} 
1 & \text{if } 0 \leq \theta < \pi \\
0 & \text{if } \pi \leq \theta \leq 2\pi 
\end{cases} \]

is what?

9. Solve the Dirichlet problem on the unit disc when the boundary function \( f(\theta) \) is given by

\[ f(\theta) = \begin{cases} 
-1 & \text{if } -\pi < \theta \leq 0 \\
1 & \text{if } 0 < x \leq \pi .
\end{cases} \]

10. Is the function

\[ u(x, y) = x^2 + y^2 \]

harmonic?

11. Write the Poisson integral formula and explain what problem it solves.

12. Explain why the Fourier series of an odd function has only sine terms.