PRACTICE THIRD MIDTERM EXAM

1. Verify that 0 is a regular singular point for the differential equation
   \[ x^2 y'' + x(x + 1)y' - y = 0. \]
   Now determine the values of \( m \) that will make the Frobenius solution valid.

2. Calculate the Fourier series of the function \( f(x) = -x, -\pi \leq x \leq \pi \).

3. Calculate the cosine series of the function \( g(x) = 2x \) on the interval \([0, \pi]\).

4. Calculate the Fourier series of the function \( h(x) = -x \) on the interval \([-3, 3]\).

5. Calculate the inverse Laplace transform of the function
   \[ F(p) = \frac{1}{p^4 - 4p^2}. \]

6. Solve the initial value problem
   \[ y'' - 2y' + y = 0, \quad y(0) = 0, \quad y'(0) = 3. \]
   using the Laplace transform.

7. Calculate the convolution of the functions \( f(t) = t \) and \( g(t) = e^{2t} \).

8. Use the unit impulse function to solve the initial value problem
   \[ y'' + y' - 6y = t, \quad y(0) = 0, \quad y'(0) = 0. \]

9. Explain why the function \( f(x) = (\cos x) \cdot (\sin x) \) is odd on the interval \([-\pi, \pi]\).
10. Define an inner product on the square integrable functions on $[-\pi, \pi]$ by
\[
\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot g(x) \, dx.
\]
Using this inner product, explain why the functions $\sin jx$ and $\cos kx$ are orthogonal for integers $j$ and $k$.

11. If $F(p)$ is the Laplace transform of the function $f$, then explain why
\[
\frac{d}{dp} F(p) = L[-xf(x)].
\]

12. Calculate
\[
L[xe^x].
\]