1. Verify that 0 is a regular singular point for the differential equation

\[ 2x^2 y'' + xy' - (x + 1)y = 0. \]

Now determine the values of \( m \) that will make the Frobenius solution valid.
2. Calculate the Fourier series of the function

\[ g(x) = \begin{cases} 
0 & \text{if } -\pi \leq x \leq 0 \\
1 & \text{if } 0 < x \leq \pi ,
\end{cases} \]

\(-\pi \leq x \leq \pi .\)
3. Calculate the sine series of the function $g(x) = \cos x$ on the interval $[0, \pi]$. 
4. Calculate the Fourier series of the function \( h(x) = x \) on the interval \([-1, 1]\).
5. Calculate the inverse Laplace transform of the function

\[ F(p) = \frac{p + 3}{p^2 + 2p + 5}. \]
6. Solve the initial value problem

\[ y'' + 2y' + 2y = 2, \quad y(0) = 0, \quad y'(0) = 1. \]

using the Laplace transform.
7. Calculate the convolution of the functions $f(t) = t$ and $g(t) = \sin t$. 
8. Use the unit impulse function to solve the initial value problem

\[ y'' + 5y' + 6y = 5e^{3t} , \quad y(0) = 0 , \quad y'(0) = 0 . \]
9. Explain why the function \( f(x) = \sin x \cdot \sin x \) is even on the interval \([-\pi, \pi]\).
10. Define an inner product on the square integrable functions on $[-\pi, \pi]$ by

$$\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot g(x) \, dx.$$ 

Using this inner product, explain why the functions $\sin jx$ and $\sin kx$ are orthogonal when $j \neq k$. 
11. If $F(p)$ is the Laplace transform of the function $f$, then explain why

$$\frac{d^2}{dp^2} F(p) = L[x^2 f(x)](p).$$
12. Calculate

\[ L[x \cos x] . \]