Math 233 Spring, 2017
Krantz March 7, 2017

Second Midterm

General Instructions: Read the statement of each problem carefully. Do only what is requested—nothing more and nothing less. Of course you need not show any work for the multiple choice or the TRUE/FALSE questions. For the questions that require a written answer, provide a *complete solution.* If you only write the answer then you will not get full credit.

Be sure to ask questions if anything is unclear. This exam is worth 100 points.

(10 points) **1.** A curve in space is given by

$$\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j} - 2t\mathbf{k}\,.$$

Calculate the velocity vector at the point $(1, 1, -2)$.

(a) $3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$

(b) $1\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$

(c) $2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$

(d) $-\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$

(e) none of the above

$(1,1,-2)$ corresponds to $t = 1$.

$\underline{\underline{r}}'(t) = 2t\,\underline{\underline{i}} + 3t^2\,\underline{\underline{j}} - 2\,\underline{\underline{k}}$

$\underline{\underline{r}}'(1) = 2\,\underline{\underline{i}} + 3\,\underline{\underline{j}} - 2\,\underline{\underline{k}}$

(10 points) **2.** What is the tangent line to the curve $\mathbf{r}(t) = (\cos t)\mathbf{i} + (\sin t)\mathbf{j} + t\mathbf{k}$ at the point $(-1, 0, \pi)$?

(a)

$$\begin{aligned} x &= -1 + 0t \\ y &= 0 - 1t \\ z &= \pi + 1t \end{aligned}$$

(b)

$$\begin{aligned} x &= -2 + 0t \\ y &= 0 - 2t \\ z &= \pi + 2t \end{aligned}$$

(c)

$$\begin{aligned} x &= 1 + 0t \\ y &= 0 + 1t \\ z &= \pi + 1t \end{aligned}$$

(d)

$$\begin{aligned} x &= -1 - 1t \\ y &= 0 + 1t \\ z &= 2\pi + 1t \end{aligned}$$

(e)

$$\begin{aligned} x &= 0 + 0t \\ y &= 1 - 1t \\ z &= 2 + 1t \end{aligned}$$

The point $(-1, 0, \pi)$ corresponds to $t = \pi$.

$\underline{r}'(t) = -\sin t\, \underline{i} + \cos t\, \underline{j} + 1\, \underline{k}$

$\underline{r}'(\pi) = 0\, \underline{i} - 1\, \underline{j} + 1\, \underline{k}$

The tangent line is then

$$\begin{aligned} x &= -1 + 0t \\ y &= 0 - 1t \\ z &= \pi + 1t \end{aligned}$$

(10 points) **3.** The length of that portion of the curve $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ between the points $(1, 0, 0)$ and $(0, 1, \pi/2)$ is

(a) $\sqrt{2\pi}$

(b) $\pi\sqrt{2}$

(c) $\sqrt{2}$

(d) $\pi\sqrt{2}/2$

(e) $2\pi\sqrt{2}$

The points correspond to $t = 0$ and $t = \pi/2$.

$$\underline{r}'(t) = -\sin t\, \underline{i} + \cos t\, \underline{j} + 1\, \underline{k}$$

$$\|\underline{r}'(t)\| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}$$

$$\text{arc length} = \int_0^{\pi/2} \|\underline{r}'(t)\|\, dt = \int_0^{\pi/2} \sqrt{2}\, dt = \sqrt{2}\,\pi/2$$

(10 points) **4.** What does Kepler's First Law say?

- **(a)** That planets travel in circular orbits.
- **(b)** That planets travel in elliptical orbits with the sun in the center of the ellipse.
- **(c)** That planets travel in elliptical orbits with the sun at one focus of the ellipse.
- **(d)** That planets travel in triangular orbits.
- **(e)** That plants travel in helical orbits.

(10 points) **5.** **TRUE** or **FALSE**: As the radius of a circle gets larger, the curvature gets smaller.

- **(a)** **TRUE**
- **(b)** **FALSE**

(10 points) **6.** The curvature of the curve $\mathbf{r}(t) = \cos t\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ at the point $(1, 0, 0)$ is

(a) 1/5
(b) 1/3
(c) 1/4
(d) 1/6
(e) 1/2

The point $(1,0,0)$ corresponds to $t=0$.

$\underline{r}'(t) = -\sin t\,\underline{i} + \cos t\,\underline{j} + 1\underline{k}$, $\underline{r}'(0) = 0\underline{i} + 1\underline{j} + 1\underline{k}$

$\underline{r}''(t) = -\cos t\,\underline{i} - \sin t\,\underline{j} + 0\underline{k}$, $\underline{r}''(0) = -1\underline{i} + 0\underline{j} + 0\underline{k}$

$$\underline{r}'(0) \times \underline{r}''(0) = \det\begin{pmatrix} \underline{i} & \underline{j} & \underline{k} \\ 0 & 1 & 1 \\ -1 & 0 & 0 \end{pmatrix} = 0\underline{i} - 1\underline{j} + 1\underline{k}$$

$$\|\underline{r}'(0) \times \underline{r}''(0)\| = \sqrt{0^2+1^2+1^2} = \sqrt{2}, \quad \|\underline{r}'(0)\| = \sqrt{0^2+1^2+1^2} = \sqrt{2}$$

$$\kappa = \frac{\|\underline{r}'(0) \times \underline{r}''(0)\|}{\|\underline{r}'(0)\|^3} = \frac{\sqrt{2}}{(\sqrt{2})^3} = \frac{1}{2}.$$

(10 points) **7.** The gradient of the function $f(x, y, z) = xyz^2$ is

(a) $\langle yz^2, xz^2, 2xyz\rangle$
(b) $\langle 2yz^2, xz^2, 2xyz\rangle$
(c) $\langle yz^2, 3xz^2, 2xyz\rangle$
(d) $\langle yz^2, -xz^2, 2xyz\rangle$
(e) $\langle yz^2, xz^2, -2xyz\rangle$

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

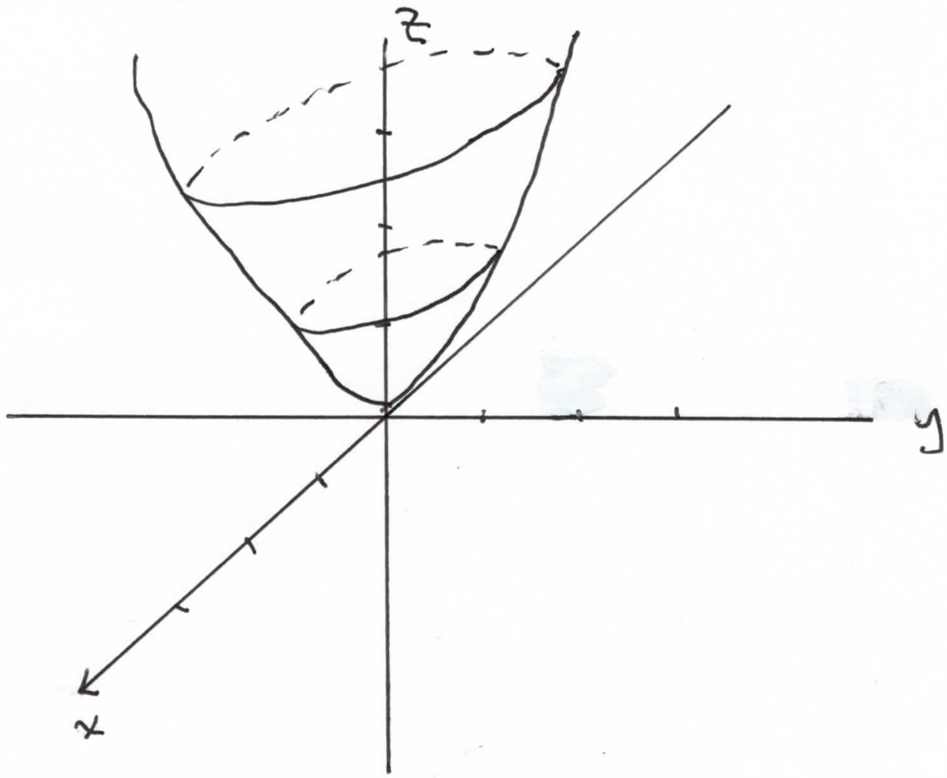
$$= \langle yz^2, xz^2, 2xyz \rangle$$

(10 points) **8.** Let $\mathbf{u} = \langle 1/\sqrt{2}, 1/\sqrt{2} \rangle$ be a direction vector. Define $f(x,y) = x \sin y$. Calculate $D_{\mathbf{u}} f$.

(a) $(1/\sqrt{2}) \cos y + (1/\sqrt{2}) x \cos y$
(b) $(2/\sqrt{2}) \sin y + (1/\sqrt{2}) x \cos y$
(c) $(1/\sqrt{2}) \sin y - (1/\sqrt{2}) x \cos y$
(d) $(1/\sqrt{2}) \sin y + (1/\sqrt{2}) x \cos y$
(e) $(1/\sqrt{3}) \sin y + (1/\sqrt{2}) x \cos y$

$$D_{\underline{u}} f = \frac{1}{\sqrt{2}} \frac{\partial f}{\partial x} + \frac{1}{\sqrt{2}} \frac{\partial f}{\partial y}$$
$$= \frac{1}{\sqrt{2}} \sin y + \frac{1}{\sqrt{2}} x \cos y$$

(10 points) **9.** On a set of x-y-z axes, sketch the graph of $f(x, y) = x^2+4y^2$. Be sure to show coordinates on your axes.



(10 points) **10.** Let $f(x, y) = x^2 \sin(xy)$. Calculate $\partial f/\partial x$ and $\partial f/\partial y$.

$$\frac{\partial f}{\partial x} = 2x\sin(xy) + x^2\cos(xy)\cdot y$$

$$\frac{\partial f}{\partial y} = x^2\cos(xy)\cdot x$$