Math 309 Spring, 2023
Krantz

PRACTICE FINAL EXAM

(12 points) **1.** Let $\mathcal{P}_5$ be the collection of all polynomials of degree not exceeding 5. Explain why this is a vector space.

(14 points) **2.** Give an explicit description of the null space of the matrix

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 2 & 1 & 3 \\ 0 & 3 & 3 \end{bmatrix} .$$

(14 points) **3.** Consider the vector space

$$\text{Span}\{\mathbf{v}_1\mathbf{v}_2, \mathbf{v}_3\}$$

where

$$\mathbf{v}_1 = \langle 2, 1, 4 \rangle ,$$
$$\mathbf{v}_2 = \langle 1, 3, 3 \rangle ,$$
$$\mathbf{v}_3 = \langle 1, -2, 1 \rangle .$$

What is the dimension of this space?

(14 points) **4.** Find a basis for the column space of

$$B = \begin{bmatrix} 0 & -2 & 4 \\ 1 & 1 & 1 \\ 6 & 0 & 0 \end{bmatrix} .$$

(14 points) **5.** Let

$$\mathbf{b}_1 = \langle 2, 0, 3 \rangle ,$$
$$\mathbf{b}_2 = \langle 3, 0, 2 \rangle ,$$
$$\mathbf{b}_3 = \langle 0, 4, 0 \rangle .$$

Write $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$. What are the coordinates of the vector $\langle 2, 1, 2 \rangle$ with respect to $\mathcal{B}$?

(14 points) **6.** Find all eigenvalues and eigenvectors of the matrix

$$\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$$

(14 points) **7.** Use any method to diagonalize the matrix

$$\begin{bmatrix} 2 & 4 \\ 4 & 2 \end{bmatrix}$$

(14 points) **8.** What is the characteristic polynomial of the matrix

$$\begin{bmatrix} 4 & 0 & 2 \\ -2 & 2 & 0 \\ 3 & 0 & 1 \end{bmatrix} ?$$

(14 points) **9.** What is the cosine of the angle between the vector $\langle 2, 1, 3 \rangle$ and the $\langle 1, 2, 4 \rangle$?

(12 points) **10.** What is the projection of the vector $\mathbf{y} = \langle 3, 1, 2 \rangle$ into the vector $\mathbf{u} = \langle 1, 1, 3 \rangle$?

(12 points) **11.** What is the distance of the point $(2, 1, 2)$ from the 2-dimensional subspace

$$W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2\}$$

with

$$\mathbf{u}_1 = \langle 3, 0, 2 \rangle \qquad \mathbf{u}_2 = \langle 2, 0, 3 \rangle \,?$$

(12 points) **12.** Are any of the vectors $\langle 2, 1, 6 \rangle$, $\langle -3, 0, 1 \rangle$, and $\langle 1, 0, 3 \rangle$ perpendicular to any of the others?

(14 points) **13.** What is a basis for the subspace of $\mathbf{R}^3$ spanned by $\langle 2, 1, 3 \rangle$, $\langle 2, 4, 1 \rangle$, and $\langle 0, -3, 2 \rangle$?

(14 points) **14.** Use the Gram-Schmidt orthogonalization method to produce an orthogonal basis for the subspace of $\mathbf{R}^3$ spanned by $\langle 1, 4, 2 \rangle$ and $\langle 6, 3, 2 \rangle$.

(12 points) **15.** Produce a vector of length 1 that is orthogonal to the vector $\langle 1, 3, 4 \rangle$ in the space $\mathbf{R}^3$.