MATH 309

SOLUTIONS TO HOMEWORK 1

7. $\left[\begin{array}{ccc|c} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{array}\right]$

The last row is $0 = 1$ which is nonsense.
So no solution.

9. $\left[\begin{array}{cccc|c} 1 & -1 & 0 & 0 & -4 \\ 0 & 1 & -3 & 0 & -7 \\ 0 & 0 & 1 & -3 & -1 \\ 0 & 0 & 0 & 0 & 4 \end{array}\right]$

The last row is $0 = 4$ so no solution.

11. $\left[\begin{array}{ccc|c} 0 & 1 & 4 & -4 \\ 1 & 3 & 3 & -2 \\ 3 & 7 & 5 & 6 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 3 & 3 & -2 \\ 0 & 1 & 4 & -4 \\ 3 & 7 & 5 & 6 \end{array}\right]$

$$\to \left[\begin{array}{ccc|c} 1 & 3 & 3 & -2 \\ 0 & 1 & 4 & -4 \\ 0 & -2 & -4 & 12 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 3 & 3 & -2 \\ 0 & 1 & 4 & -4 \\ 0 & 0 & 4 & 4 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 3 & 3 & -2 \\ 0 & 1 & 4 & -4 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$x_3 = 1$

$x_2 + 4x_3 = -4 \Rightarrow x_2 = -4x_3 - 4 = -8$

$x_1 + 3x_2 + 3x_3 = -2$

$x_1 = -3x_2 - 3x_3 - 2 = (3)(-8) - 3 \cdot 1 - 2$

$= 24 - 3 - 2 = 19$

So $x_1 = 19, x_2 = -8, x_3 = 1$

13. $\left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 2 & 2 & 9 & 7 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{array}\right]$

$\rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{array}\right]$

So $x_3 = -1$

$x_2 = -2 - 5x_3 = -2 + 5 = 3$

$x_1 = 3x_3 + 8 = -3 + 8 = 5$

14.

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ -1 & 1 & 5 & 2 \\ 0 & 1 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & -2 & 5 & 7 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & -2 & 5 & 7 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 7 & 7 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -3 & 0 & 5 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$x_3 = 1$

$x_2 = -x_3 = -1$

$x_1 = 3x_2 + 5 = 2$

17. $x_1 = 5,\ x_2 = 3,\ x_3 = -1$

$x_1 - 3x_3 = 5 + 3 = 8$ ✓

$2x_1 + 2x_2 + 9x_3 = 10 + 6 - 9 = 7$ ✓

$x_2 + 5x_3 = 3 - 5 = -2$ ✓

19. $\left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & -2 & 3 & 2 & 1 \\ 3 & 0 & 0 & 7 & -5 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 3 & 0 & 2 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 3 & 0 & 0 & 7 & -5 \end{array}\right]$

$\rightarrow \left[\begin{array}{cccc|c} 3 & 0 & 0 & 7 & -5 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 1 & 0 & 3 & 0 & 2 \end{array}\right]$

$\rightarrow \left[\begin{array}{cccc|c} 3 & 0 & 0 & 7 & -5 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & 3 & -\frac{7}{3} & 11/3 \end{array}\right]$

$\rightarrow \left[\begin{array}{cccc|c} 3 & 0 & 0 & 7 & -5 \\ 0 & 1 & 0 & -3 & 3 \\ 0 & 0 & 3 & -4 & 7 \\ 0 & 0 & 0 & 5/3 & -10/3 \end{array}\right]$

consistent

20. $\left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ -2 & 3 & 2 & 1 & 5 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 3 & 2 & -3 & -1 \end{array}\right]$

$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & -1 & -3 & -1 \end{array}\right]$

$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & -2 & -3 \\ 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ consistent

23. $\left[\begin{array}{cc|c} 1 & h & 4 \\ 3 & 6 & 8 \end{array}\right]$

Take $h = 4$

$\left[\begin{array}{cc|c} 1 & 4 & 4 \\ 3 & 6 & 8 \end{array}\right]$

$\rightarrow \left[\begin{array}{cc|c} 1 & 4 & 4 \\ 0 & -6 & -4 \end{array}\right]$

$\rightarrow \left[\begin{array}{cc|c} 1 & 4 & 4 \\ 0 & 1 & 2/3 \end{array}\right]$ consistent

25. $\left[\begin{array}{cc|c} 1 & 3 & -2 \\ -4 & h & 8 \end{array}\right]$

Take $h = -12$

$\left[\begin{array}{cc|c} 1 & 3 & -2 \\ -4 & -12 & 8 \end{array}\right]$

$\rightarrow \left[\begin{array}{cc|c} 1 & 3 & -2 \\ 0 & 0 & 0 \end{array}\right]$ consistent.