

MATH 309

①

SOLUTIONS TO HW 2

1. a) reduced

c) reduced

2. b) reduced

d) not reduced

$$3. \begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & -5 & -10 & -15 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -6 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \textcircled{1} & 2 & 3 & 4 \\ 0 & \textcircled{-3} & -6 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

↑ ↑
pivot columns

$$5. \begin{bmatrix} \boxed{x} & \times \\ 0 & \boxed{x} \end{bmatrix} \quad \begin{bmatrix} \boxed{x} & * \\ 0 & 0 \end{bmatrix}$$

$$11. \left[\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_3 \text{ free} = \lambda$$

$$x_2 \text{ free} = \mu$$

$$3x_1 - 4\mu + 2\lambda = 0$$

$$x_1 = \frac{4}{3}\mu - \frac{2}{3}\lambda$$

$$22. \left[\begin{array}{ccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & -4 & 8 & 12 \end{array} \right]$$

$$\rightarrow \left[\begin{array}{ccc|c} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

✓ free = λ x_1 free = μ

②

$$x_3 = 2\lambda - 3$$

$$x_1 = 7x_2 - 6x_4 + 5$$

$$= 7\mu - 6\lambda + 5$$

$$13. \left[\begin{array}{cccc|c} 1 & -3 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -4 \\ 0 & 0 & 0 & 1 & 9 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right] \rightarrow$$

$$x_5 \text{ free} \rightarrow \lambda$$

$$x_4 = -9\lambda + 4$$

$$x_3 \text{ free} \rightarrow \mu$$

$$x_2 = 4\lambda + 1$$

$$x_1 = 3x_2 + x_4 - 2 = 3(4\lambda + 1) + (-9\lambda + 4) - 2$$

$$= 3\lambda + 5$$

$$14. \left[\begin{array}{cccc|c} 1 & 2 & -5 & -4 & -5 \\ 0 & 1 & -6 & -4 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$x_4 \text{ free} \rightarrow \lambda$$

$$x_3 \text{ free} \rightarrow \mu$$

$$x_5 = 0$$

$$x_2 = 6x_3 + 4x_4 + 2 = 6\mu + 4\lambda + 2$$

$$x_1 = -2x_2 + 5x_3 + 4x_4 - 5 = -2(6\mu + 4\lambda + 2) + 5\mu + 4\lambda - 5$$

$$= -7\mu - 4\lambda - 9$$

$$17. \begin{aligned} 3x_1 - 4x_2 + 2x_3 &= 0 & 3\left(\frac{4}{3}\mu - \frac{2}{3}\lambda\right) - 4\mu + 2\lambda &= 0 \quad \checkmark \\ -9x_1 + 12x_2 + 6x_3 &= 0 & -9\left(\frac{4}{3}\mu - \frac{2}{3}\lambda\right) + 12\mu - 6\lambda &= 0 \quad \checkmark \\ -6x_1 + 8x_2 - 4x_3 &= 0 & -6\left(\frac{4}{3}\mu - \frac{2}{3}\lambda\right) + 8\mu - 4\lambda &= 0 \quad \checkmark \end{aligned}$$

21. $\left[\begin{array}{cc|c} 2 & 3 & 7/2 \\ 4 & 6 & 7 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 2 & 3 & 7/2 \\ 0 & 0 & 0 \end{array} \right]$

x_2 free λ

$$2x_1 + 3\lambda = \frac{7}{2}$$

$$x_1 = -\frac{3}{2}\lambda + \frac{7}{4}$$

22. $\left[\begin{array}{cc|c} 1 & -3 & -2 \\ 5 & 1 & -7 \end{array} \right] \rightarrow \left[\begin{array}{cc|c} 1 & -3 & -2 \\ 0 & 16 & 3 \end{array} \right]$

$$x_2 = \frac{3}{16}$$

$$x_1 - 3 \cdot \frac{3}{16} = -2$$

$$x_1 = \frac{9}{16} - 2 = \frac{-23}{16}$$

34. I