MATH 309

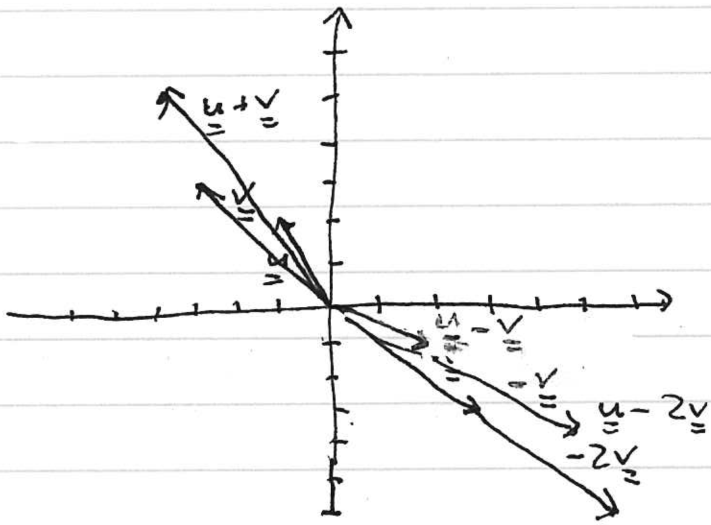
SOLUTIONS TO HOMEWORK 3

1. $\underline{\underline{u}} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, $\underline{\underline{v}} = \begin{bmatrix} -3 \\ 3 \end{bmatrix}$

$$\underline{\underline{u}} + \underline{\underline{v}} = \begin{bmatrix} -4 \\ 5 \end{bmatrix}$$

$$\underline{\underline{u}} - 2\underline{\underline{v}} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} - \begin{bmatrix} -6 \\ 6 \end{bmatrix} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$$

3.



5.

$$6x_1 - 3x_2 = 1$$
$$-1x_1 + 4x_2 = -7$$
$$5x_1 + 0x_2 = -5$$

11.

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 2 & 8 & 6 \\ -2 & 1 & 6 & -1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 2 & 8 & 6 \\ 0 & 1 & 16 & 3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 0 & -24 & 0 \\ 0 & 1 & 26 & 3 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 16 & 3 \\ 0 & 0 & -24 & 0 \end{array}\right] \qquad \begin{array}{l} x_3 = 0 \\ x_2 = 3 \\ x_1 = 2 \end{array}$$

Yes, $\underline{\underline{b}}$ is a linear combination of the given vectors.

12. $\left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ -2 & 5 & 0 & 11 \\ 2 & 5 & 8 & -7 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 5 & 4 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 2 & -5 \\ 0 & 5 & 4 & 1 \\ 0 & 0 & 0 & 2 \end{array}\right]$

inconsistent

So $\underline{b}$ is <u>not</u> a linear combination of the given vectors.

13. $\left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{array}\right]$

inconsistent

So $\underline{b}$ is <u>not</u> a linear combination of the columns.

14. $\left[\begin{array}{ccc|c} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 11 & -2 \end{array}\right]$

$x_3 = -\frac{2}{11}$

$x_2 = -\frac{41}{33}$

$x_1 = \frac{245}{33}$

So $\underline{b}$ <u>is</u> a linear combination of the columns

17. Consider $x_1 \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$

We can solve for x_1, x_2:

$x_1 = -2, x_2 = -3$

Then $h = -2x_1 + 7x_2 = 4 - 21 = -17.$

21. $\left[\begin{array}{cc|c} 2 & 2 & h \\ -1 & 1 & k \end{array}\right] \to \left[\begin{array}{cc|c} 2 & 2 & h \\ 0 & 2 & k+\frac{1}{2}h \end{array}\right] \to \left[\begin{array}{cc|c} 2 & 0 & \frac{1}{2}h - k \\ 0 & 2 & k+\frac{1}{2}h \end{array}\right]$

So $x_1 = \frac{1}{4}h - \frac{1}{2}k$

$x_2 = \frac{1}{2}k + \frac{1}{4}h.$

$x_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} h \\ k \end{bmatrix}$

22. $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ The vector $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is

not in the span of the columns because

$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is not a multiple of $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

24. TRUE