

## SOLUTIONS TO HW 5

§ 2.1

$$1. A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -3 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}$$

$$-2A = \begin{bmatrix} -4 & 0 & 2 \\ -8 & 6 & -4 \end{bmatrix} \quad B - 2A = \begin{bmatrix} 3 & -5 & 3 \\ -7 & 2 & -7 \end{bmatrix}$$

$AC$  is not defined

$$CD = \begin{bmatrix} 1 & 13 \\ -7 & -6 \end{bmatrix}$$

$$3. 3I_2 - A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -5 & 5 \end{bmatrix}$$

$$(3I_2)A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 4 & -1 \\ 5 & -2 \end{bmatrix} = \begin{bmatrix} 12 & -3 \\ 15 & -6 \end{bmatrix}.$$

$$5. A \underset{=1}{b} = \begin{bmatrix} -7 \\ 7 \\ 12 \end{bmatrix} \quad A \underset{=2}{b} = \begin{bmatrix} 6 \\ -16 \\ -11 \end{bmatrix} \Rightarrow AB = \begin{bmatrix} -7 & 6 \\ 7 & -16 \\ 12 & -11 \end{bmatrix}$$

$$AB = \begin{bmatrix} -7 & 6 \\ 7 & -16 \\ 12 & -11 \end{bmatrix} \cdot \begin{bmatrix} 3 & -4 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 6 \\ 7 & -16 \\ 12 & -11 \end{bmatrix}$$

7.  $B$  is  $3 \times 7$

8. 3 rows

$$10. AB = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix} \begin{bmatrix} 84 \\ 55 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & -3 \\ -4 & 6 \end{bmatrix} \begin{bmatrix} 5 & -2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -7 \\ -2 & 14 \end{bmatrix}$$

But  $B \neq C$ .

§2.2

$$1. \left[ \begin{array}{cc|cc} 8 & 3 & 1 & 0 \\ 5 & 2 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cc|cc} 8 & 3 & 1 & 0 \\ 0 & 1/8 & -5/8 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cc|cc} 8 & 3 & 1 & 0 \\ 0 & 1 & -5 & 8 \end{array} \right] \\ \rightarrow \left[ \begin{array}{cc|cc} 8 & 0 & 16 & -24 \\ 0 & 1 & -5 & 8 \end{array} \right] \rightarrow \left[ \begin{array}{cc|cc} 1 & 0 & 2 & -3 \\ 0 & 1 & -5 & 8 \end{array} \right]$$

So the inverse matrix is  $\begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix}$

$$3. \left[ \begin{array}{cc|cc} 8 & 3 & 1 & 0 \\ -7 & -3 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cc|cc} 8 & 3 & 1 & 0 \\ 0 & -3/8 & 7/8 & 1 \end{array} \right] \\ \rightarrow \left[ \begin{array}{cc|cc} 8 & 3 & 1 & 0 \\ 0 & -3 & 7 & 8 \end{array} \right] \rightarrow \left[ \begin{array}{cc|cc} 8 & 0 & 8 & 8 \\ 0 & -3 & 7 & 8 \end{array} \right] \\ \rightarrow \left[ \begin{array}{cc|cc} 1 & 0 & 1 & 1 \\ 0 & 1 & -7/3 & -8/3 \end{array} \right]$$

So inverse matrix is  $\begin{bmatrix} 1 & 1 \\ -7/3 & -8/3 \end{bmatrix}$ .

$$5. \begin{bmatrix} 8 & 3 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$7. \text{ let } \underline{b} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\underline{x} = A^{-1} \underline{b} = \begin{bmatrix} 2 & -3 \\ -5 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ -18 \end{bmatrix}$$

$$9a) \left[ \begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 5 & -12 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 2 & -5 & 1 \end{array} \right]$$

$$\rightarrow \left[ \begin{array}{cc|cc} 1 & 0 & 6 & -1 \\ 0 & 2 & -5 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{cc|cc} 1 & 0 & 6 & -1 \\ 0 & 1 & -5/2 & 1/2 \end{array} \right]$$

$$A \underline{x} = \underline{b}_1 \quad \underline{x} = A^{-1} \underline{b}_1 = \begin{bmatrix} 6 & -1 \\ -5/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -9 \\ 4 \end{bmatrix}$$

$$A \underline{x} = \underline{b}_2 \quad \underline{x} = A^{-1} \underline{b}_2 = \begin{bmatrix} 6 & -1 \\ -5/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1 \\ -5 \end{bmatrix} = \begin{bmatrix} 21 \\ -5 \end{bmatrix}$$

$$A \underline{x} = \underline{b}_3 \quad \underline{x} = A^{-1} \underline{b}_3 = \begin{bmatrix} 6 & -1 \\ -5/2 & 1/2 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

$$A \underline{x} = \underline{b}_4 \quad \underline{x} = A^{-1} \underline{b}_4 = \begin{bmatrix} 6 & -1 \\ -5/2 & 1/2 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 13 \\ -5 \end{bmatrix}$$

$$20. \quad AD = I$$

$$A^{-1}(AD) = A^{-1}I = A^{-1}$$

$$(A^{-1}A)D = A^{-1}$$

$$I \cdot D = A^{-1}$$

$$D = A^{-1}$$