Math 309 Spring, 2023
Krantz

FIRST PRACTICE MIDTERM EXAM

(12 points) **1.** Use an augmented matrix to solve this system of linear equations.

$$\begin{aligned} x_1 - 3x_2 &= 5 \\ -x_1 + x_2 + 5x_3 &= 2 \\ x_2 + x_3 &= 0 \end{aligned}$$

(12 points) **2.** Find the general solution of the system whose augmented matrix is give here.

$$\left[\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{array}\right]$$

(12 points) **3.** Determine whether $\mathbf{b}$ is a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \ \mathbf{v}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \ \mathbf{v}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$$

(12 points) **4.** Given A and $\mathbf{b}$, write the augmented matrix for the linear system that corresponds to the matrix equation $A\mathbf{x} = \mathbf{b}$. Then solve the system and write the solution as a vector.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 5 \\ -2 & -4 & -3 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} -2 \\ 2 \\ 9 \end{bmatrix}$$

(14 points) **5.** Write the solution of the given homogeneous system in paremetric vector form (using λ, μ, etc.).

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 0 \\ -4x_1 - 9x_2 + 2x_3 &= 0 \\ -3x_2 - 6x_3 &= 0 \end{aligned}$$

(14 points) **6.** Determine whether the columns of this matrix form a linearly independent set.

$$\begin{bmatrix} 0 & -8 & 5 \\ 3 & -7 & 4 \\ -1 & 5 & -4 \\ 1 & -3 & 2 \end{bmatrix}$$

(12 points) **7.** Given the matrix A, define the linear mapping T by $T\mathbf{x} = A\mathbf{x}$. Find a vector $\mathbf{x}$ whose image under T is $\mathbf{b}$. Say whether or not $\mathbf{x}$ is unique.

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -2 & 1 & 6 \\ 3 & -2 & -5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -1 \\ 7 \\ -3 \end{bmatrix}$$

(12 points) **8.** The map T from $\mathbf{R}^2$ to $\mathbf{R}^4$ satisfies $T(\mathbf{e}_1) = (2, 1, 2, 1)$, $T(\mathbf{e}_2) = (-5, 2, 0, 0)$. [Here $\mathbf{e}_1$, $\mathbf{e}_2$ are the columns of the 2×2 identity matrix.] Find the standard matrix of T.