MATH 309

SOLUTIONS TO PRACTICE MIDTERM 2

1. $$\left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 2 & 0 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & -\frac{1}{2} & -\frac{1}{2} & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & -2 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 0 & -4 & 2 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{1}{2} & -2 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & -2 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 4 & -2 \end{array}\right].$$

So inverse matrix is $\begin{bmatrix} 0 & -2 & 1 \\ 0 & 1 & 0 \\ 1 & 4 & -2 \end{bmatrix}$.

2. $$\left[\begin{array}{ccc|c} 1 & -3 & -4 & 1 \\ -4 & 6 & -2 & 1 \\ -3 & 7 & 6 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -3 & -4 & 1 \\ 0 & -6 & -18 & 5 \\ 0 & -2 & -6 & 4 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & -3 & -4 & 1 \\ 0 & -6 & -18 & 5 \\ 0 & 0 & 0 & 7/3 \end{array}\right]$$

Inconsistent. So

$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ is <u>not</u> in the column space.

3. $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & 0 \end{array}\right]$

$$x_3 = 0$$
$$x_2 = 0$$
$$x_1 = -2x_3 = 0$$

So the null space is just the $\underline{0}$ vector.

4. $\left[\begin{array}{ccc|c} 0 & -3 & 4 & 0 \\ 1 & 1 & -1 & 0 \\ 8 & 0 & 0 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3 & 4 & 0 \\ 8 & 0 & 0 & 0 \end{array}\right]$

$\to \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3 & 4 & 0 \\ 0 & -8 & 8 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -3 & 4 & 0 \\ 0 & 0 & -8/3 & 0 \end{array}\right]$

$$x_3 = 0$$
$$x_2 = 0$$
$$x_1 = 0$$

So the columns are linearly independent. Hence a basis for the column space is

$$\begin{bmatrix} 0 \\ 1 \\ 8 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$$

5. $\det \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 1 \\ 4 & 1 & 4 \end{bmatrix} = 1\,(1\cdot 4 - 1\cdot 1) - 2\,(3\cdot 4 - 4\cdot 1) + 3\,(3\cdot 1 - 4\cdot 1)$
$= 3 - 2\cdot 8 + 3\cdot(-1) = -16.$

6. $$x_1 = \frac{\det\begin{bmatrix} 6 & -1 \\ 8 & 2 \end{bmatrix}}{\det\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}} = \frac{12+8}{4-3} = 20$$

$$x_2 = \frac{\det\begin{bmatrix} 2 & 6 \\ -3 & 8 \end{bmatrix}}{\det\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}} = \frac{16+18}{1} = 34$$

7. $$AB = \begin{bmatrix} 5 & 1 \\ 3 & 3 \end{bmatrix}\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 23 & 7 \\ 21 & 9 \end{bmatrix}$$

$$\det(AB) = \det\begin{bmatrix} 23 & 7 \\ 21 & 9 \end{bmatrix} = 23 \cdot 9 - 21 \cdot 7$$
$$= 207 - 147 = 60$$

$$\det A = \det\begin{bmatrix} 5 & 1 \\ 3 & 3 \end{bmatrix} = 15 - 3 = 12$$

$$\det B = \det\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix} = 8 - 3 = 5$$

$$60 = 12 \cdot 5.$$

8. $$\left[\begin{array}{ccc|c} 5 & -3 & 1 & 0 \\ -3 & 1 & -3 & 0 \\ 9 & -5 & 5 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 5 & -3 & 1 & 0 \\ 0 & -4/5 & -12/5 & 0 \\ 0 & 2/5 & 16/5 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 5 & -3 & 1 & 0 \\ 0 & -4/5 & -12/5 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right]$$

$$x_3 = 0$$
$$x_2 = 0$$
$$x_1 = 0$$

So only the trivial solution. Hence $\underline{v}_1, \underline{v}_2, \underline{v}_3$ are linearly <u>independent</u>.