Chapter 8

30. Let \( E_1, E_2, \ldots, E_k \) be compact. If \( \mathcal{U} = \{ U_\alpha \} \) is an open covering of \( \bigcup_j E_j \), then choose a finite subcovering \( \mathcal{U}_1 \) for \( E_1 \), choose a finite subcovering \( \mathcal{U}_2 \) for \( E_2 \), \ldots choose a finite subcovering \( \mathcal{U}_k \) for \( E_k \). Then \( \mathcal{V} = \bigcup_{j=1}^k \mathcal{U}_j \) is a finite subcovering for the union of the \( E_j \).

A finite intersection is the same idea but even easier.

31. The finite subcover consists of \((-1/4, 1/4), (5/6, 7/6)\), and the interval \((1/10, 9/10)\).

33. Let
\[
f(x) = \begin{cases} 
0 & \text{if } x \leq 1 \\
(1/2)(x - 1) & \text{if } 1 < x < 3 \\
1 & \text{if } 3 \leq x.
\end{cases}
\]

Chapter 11

1. Define \((g, h) \cdot (g', h') = (g \cdot g', h \cdot h')\). If \( G \) and \( H \) are abelian then \( G \times H \) will also be abelian.

2. The group \( G \) has only one element of order 2, and that is 2. Namely \( 2 + 2 = 4 \). But \( H \) has at least two elements of order 2, namely \((2,0)\) and \((0,2)\). So the groups cannot be isomorphic.

4. Call the group elements \( X, Y, Z \). Then we can define a group by
\[
X \cdot X = X, \\
X \cdot Y = Y \cdot X = Y,
\]
This is the same as $\mathbb{Z}_3$. Any other three-element group that you cook up will be isomorphic to this one.

10. $S$ is closed under addition. The first $T$ is not. $S$ is closed under multiplication, and so is the first $T$.

The second $T$ is closed under addition. It is also closed under multiplication.

12. The integers $\mathbb{Z}$ equipped with addition is an infinite cyclic group. The group $\mathbb{Z}_3$ with addition is a finite cyclic group.

23. Let $k = 2$. We see that

\[
\begin{align*}
[\phi(g)]^2 &= \phi(g) \cdot \phi(g) \\
&= \phi(g \cdot g) \\
&= \phi(e_G) \\
&= e_H.
\end{align*}
\]

The general case follows by induction.