Solutions to First Homework

1. b) \[
\begin{array}{|c|c|c|c|c|}
\hline
S & T & S \lor T & S \land T & (S \lor T) \Rightarrow (S \land T) \\
\hline
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T & F & T & F & F \\
T & F & T & F & F \\
F & T & T & F & F \\
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\hline
\end{array}
\]

1. d) \[
\begin{array}{|c|c|c|c|c|}
\hline
S & T & S \Rightarrow T & S \Rightarrow (S \Rightarrow T) & S \Rightarrow (S \Rightarrow (S \Rightarrow T)) \\
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\hline
\end{array}
\]

2. b) \(U \Rightarrow (V \lor V \sim S)\)

f) \(S \Rightarrow \sim T\)

3. b) Either the pie is in the sky or both some men are fools and I do have two brain cells to rub together.

d) If all politicians are honest, then either all politicians are honest or I don’t have two brain cells to rub together.
4. d) Converse: If mass exists, then I am not a fool.

Contrapositive: If mass does not exist, then I am a fool.

f) Converse: If there is peace in the world then all people should disarm.

Contrapositive: If there is no peace in the world then some people should not disarm.

5. b) True because hypothesis and conclusion are both true.

f) True because hypothesis and conclusion are both false.

6. b) \( \sim(S \lor T) \)

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d) \( \neg (\neg S \lor \neg (\neg T \lor S)) \)

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9. b) Let \( Q \) = all polynomial equations

\[ \exists q \in Q \exists x : q = ax^2 + bx + c \land a, b, c \in \mathbb{R} \land \neg \exists y \in \mathbb{R} : q(y) = 0. \]

d) \( \forall x \in \mathbb{R} \forall y \in \mathbb{R}, x \cdot y \neq 0 \Rightarrow x^2 + y^2 > 0. \)

10. b) There exists a variable with the property that, if it is exceeded by another variable, then the other variable is greater than five.

e) There is a variable so that it is not true that if the square of the variable is positive then the variable is positive.
13 b) Either men do not eat oats or does do not eat oats.

17. We need to check that each connective has a logically equivalent expression using V and ∨.

\[ A \land B \iff \neg (A \lor B) \quad \text{de Morgan} \]

\[ A \lor B \quad \text{OK} \]

\[ \neg A \quad \text{OK} \]

\[ A \Rightarrow B \iff \neg A \lor B \quad \text{done in class} \]

\[ A \iff B \iff (\neg A \lor B) \land (\neg B \lor A) \]

\[ \iff \neg (\neg (A \lor B) \lor \neg (\neg B \lor A)) \]