PRACTICE EXAM FOR FIRST MIDTERM

(12 points) 1. Write the converse and the contrapositive of the sentence

*If down is up, then life goes on.*

Label each one.

**Converse:** If life goes on, then down is up.

**Contrapositive:** If life does not go on, then

down is not up.
(12 points) 2. Are the statements $A \lor B$ and $\sim A \land B$ logically equivalent? Why or why not?

<table>
<thead>
<tr>
<th>$A$</th>
<th>$B$</th>
<th>$\sim A$</th>
<th>$A \lor B$</th>
<th>$\sim A \land B$</th>
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not the same, so not logically equivalent.

(12 points) 3. Express the statement $\forall x, \sim P(x)$ using $\exists$ instead of $\forall$.

$\sim \exists x, P(x)$. 
(12 points) 4. Prove that the square of an odd integer is odd.

Let \( k = 2n + 1 \), then

\[
\begin{align*}
    k^2 &= (2n+1)^2 = 4n^2 + 4n + 1 \\
      &= 2(2n^2 + 2n) + 1 \\
\end{align*}
\]

which is odd.

(14 points) 5. Prove that the integer 6 does not have a rational square root.

Assume to the contrary that 6 does have a rational square root \( \frac{p}{q} \) in lowest terms.

Then \( \left( \frac{p}{q} \right)^2 = 6 \Rightarrow p^2 = 6q^2 \). So 6 divides \( \text{LHS} \) hence 6 divides LHS. It follows that 6 divides \( \text{RHS} \). Hence \( 6 | p \). So \( p = 6k \). Hence

\[
(6k)^2 = 6q^2 \Rightarrow 6k^2 = q^2 \Rightarrow 6 | q^2 \Rightarrow 6 | q.
\]

We see that \( p,q \) have the common factor 6. Contradiction.
(14 points) 6. Use mathematical induction to prove that the sum of the first $n$ odd integers is $n^2$.

\[
P(n) \text{ is } 1 + 3 + 5 + \cdots + (2n-1) = n^2.
\]

\[P(1) \text{ is obviously true.}
\]

Assume $P(n)$, so $1 + 3 + 5 + \cdots + (2n-1) = n^2$.

Then $1 + 3 + 5 + \cdots + (2n-1) + (2n+1) = n^2 + 2n + 1 = (n+1)^2$.

That is $P(n+1)$.

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(12 points) 7. Give a truth table for the statement $(A \lor \sim B) \Rightarrow (\sim A \land B)$.

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<tr>
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<th>$\sim A$</th>
<th>$\sim B$</th>
<th>$A \lor \sim B$</th>
<th>$\sim A \land B$</th>
<th>$(A \lor \sim B) \Rightarrow (\sim A \land B)$</th>
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(12 points) 8. Use any method to prove that $2^k > 1 + 2k$ for $k > 2$.

$k = 3 \Rightarrow 2^3 > 1 + 2 \cdot 3 \Leftrightarrow 8 > 7$ true.

$P(k): 2^k > 1 + 2k$.

Assume $P(k)$.

$2^k > 1 + 2k$

$2^{k+1} > 2 + 4k$

We need $2 + 4k > 1 + 2(k+1)$

$2 + 4k > 1 + 2k + 2$

$2 + 4k > 2k + 3$

$2k > 1$ true.